ACReS

Academic & Career Readiness Skills

Two Plus You

Unit 2

National PASS Center

2013

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Genesee Valley Educational Partnership
success stories written here

Lesson 1

Working with Place Value Integers and Absolute Value

Words to know:

- √ **place value**
- √ **integers**
- √ **number line**
- √ **absolute value**

Sometimes, math uses very large numbers. There is a system for writing and reading these big numbers. Place value is used to read large numbers. It shows how much a digit is worth according to where it is in the number.

✓ The value of a digit in a number depends on where it is in the number. This position is called the **place value**.

Example: In the number 357, the 3 is in the *hundreds place*, 5 is in the *tens place* and 7 is in the *ones place*. This number is written and said as, "three hundred fifty seven." But still, what about <u>really</u> large numbers?

In general, very large numbers are grouped in sets of three digits. Each set of three digits is separated by a comma. This method makes the number easier to see. It also helps to write and say the number. Look at the number below. Notice where the commas have been placed.

Example: 23**,**463**,**245**,**978**,**031

Start on the <u>right</u> and count by three. The first group of three digits is hundreds. The second group of three is thousands. The third group of three is millions. The fourth group is billions. The last group is trillions!

✓ Each group of three digits is called a **period**. Commas separate each period from another.

The value of a single digit depends on its position in a number. Where a digit is within a period gives it value, too. Look at the chart that follows. The big number has been written in it by period.

Trillions period			**Billions period**			**Millions Period**			**Thousands Period**			**Unit Period**		
0	2	3	4	6	3	2	4	5	9	7	8	0	3	1
Hundred Trillions	Ten Trillions	Trillions	Hundred Billions	Ten Billions	Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones

To figure out the periods, always start at the far right. After you have figured out the periods, you are ready to write the number with words. Always write a large number starting from the far left.

In words, the number 23,463,245,978,031 is written as "twenty-three trillion, four hundred sixty-three billion, two hundred forty-five million, nine hundred seventy-eight thousand, thirty-one." Never use the word "and" when writing out or saying a number in words.

Here is the rule to write and say large numbers.

Rule to write out a number:

1. Begin at the far right and put a comma after every 3 digits.
2. Start at the right again. Count the first group as the unit period. The second group is the thousands. The third is the millions. Do this until you have reached the final group at the far left.
3. Start at the far left. Write the numbers as words. Use terms from the Units Period.
4. Write the value of the period (trillion, billion, million, etc.) followed by a comma.
5. Repeat this until you have written the values from the Unit Period.

Now you try!

1. Write the following numbers with words and say them out loud:

a. 1,345

b. 456,210

c. 1,948,111,985

d. 1,000,043,000,005

2. The population of the world is six billion, eight hundred twelve million, thirty-four thousand, three hundred ninety three. Write this with numbers.

____

3. In the number 836,204,124,385,685

a. What is the place value of the zero? ____________________________

b. What is the period value of the digits 124? ________________________

4. In each of the following, circle or highlight the digit in the place value written on the right.

a.	1,234,567	ten thousands
b.	947,183,208,264,900	hundred millions
c.	608,574	tens
d.	917,333,273,194,732	trillions
e.	1,532	ones
f.	622,948	thousands

5. Write the following numbers with words.

a. 1,009 ____

____

b. 13,076 ____

____

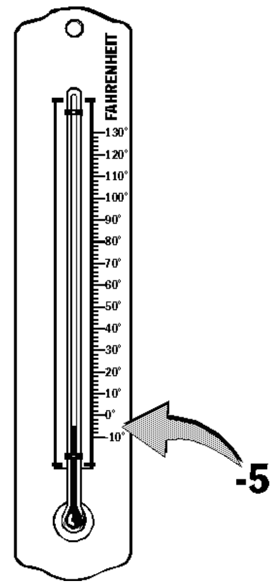
c. 100,000,000,000,000 ____

____

d. 847,256,958,123,732 ____

____

Integers and Absolute Value



One morning you wake up and look out your window. There is a blizzard! You check the thermometer. The temperature reads –5 degrees. You look closer at your thermometer. You notice that this number is below zero.

When we count, we usually use **whole numbers**.

✓ **Whole numbers** are the numbers 0, 1, 2, 3, 4, 5, 6, 7, …

You may count that you have four pencils. You may see that there are zero eggs in your refrigerator. The smallest whole number is zero. There is no largest whole number. You can always count higher than any number you can think of.

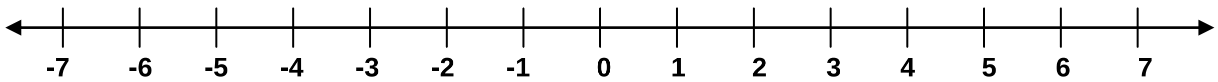
Sometimes you need to count things that are less than zero. For instance, Tiger Woods scored –1 in the 2008 PGA Golf Tournament. The temperature in the beginning of the story was –5 degrees. Numbers less than zero are called **negative numbers**.

✓ **Negative numbers** are all the numbers that are less than zero.
–1, –2, –3, –4, –5, –6,…

The group of all positive and all negative whole numbers together, plus zero, is called **integers**.

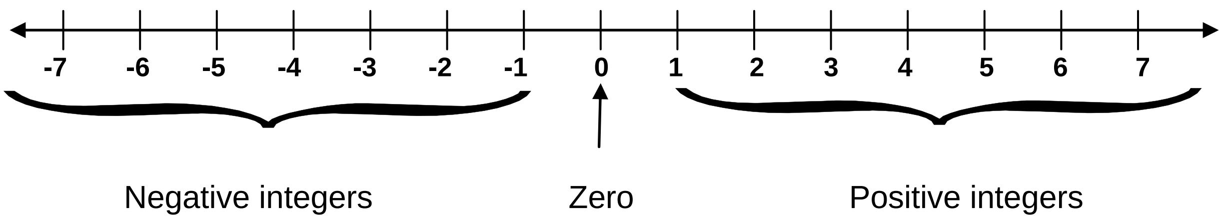
Zero is neither positive nor negative. It is just zero! The equator separates the northern and southern hemispheres of the Earth. Zero acts the same way. It separates the positive and negative integers.

One way to understand integers is to see them on a **number line**.

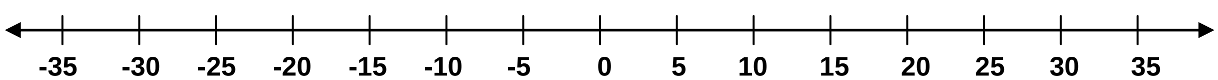


The number line is a way to show every single number. Notice that there are spaces between the integers. They show that there are numbers between the integers. We will talk about these numbers later. For now, look only at the integers.

The groups of numbers that make up integers are shown with brackets on the number line below.

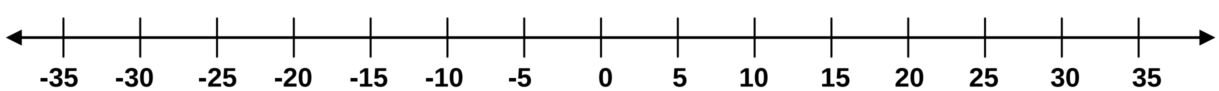


Number lines do not always show every integer. For example, look at this number line.

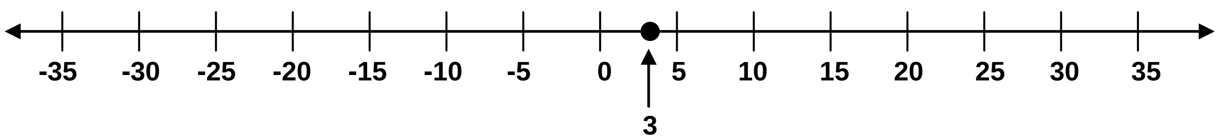


Here, the number line increases by 5 every time. As long as you always increase by the same amount, you may label your number line however you like. Not every integer is labeled, but they are still there on the line. Remember: Number lines show every number, even if they are not labeled.

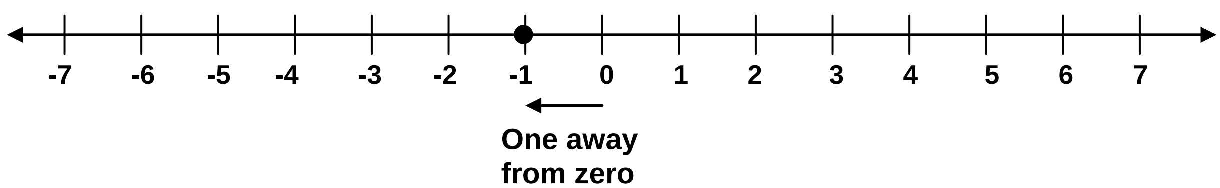
Example: Show where 3 is on the given number line.



Solution: We know that 3 will lie between 0 and 5. We also know that 3 is 3 integers away from zero, and only 2 integers away from 5. So, 3 is closer to 5 than it is to 0. Because of this, we will put a point that is slightly closer to 5 than to zero on the number line.



Notice that zero is right in the middle of the number line. It separates the negative integers from the positive integers. Zero is a very important number in math. We use it to compare with other numbers. For instance, Tiger Woods' golf score of -1 is one away from zero. You can see this on the next number line.

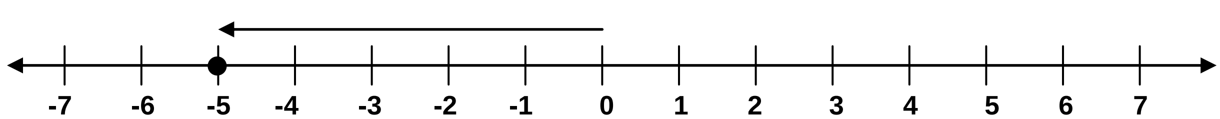


First, put a point on the line where the actual number is. Then, count how far away it is from zero. You are finding the <u>distance</u> from zero. If the number is positive, count to the right. If the number is negative, count to the left.

As you can see, –1 is one place away from zero.

Example: Show –5 on the number line, and then determine its distance from zero.

Solution: On the number line, put a point on –5 to show where it is.



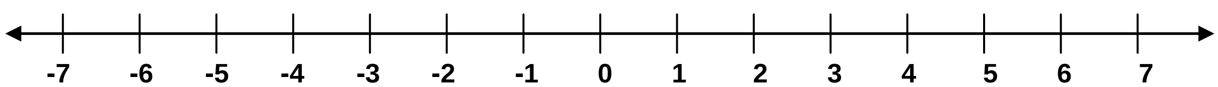
Then, using an arrow, count how many integers it is away from zero. It is 5 away from zero. To show this we say that $|-5| = 5$.

✓ The distance a number is from zero is its **absolute value**. The absolute value of a number is shown by vertical lines put to the left and right of the number. Thus, to show the absolute value of -12, we say $|-12| = 12$. The absolute value of a number is always positive.

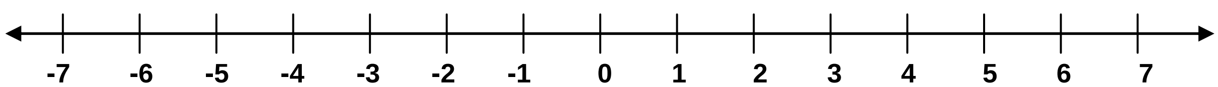
Now you try!

6. Plot the following on the number line. Find how far each number is from zero. Then, state in math writing what the absolute value of the number is.

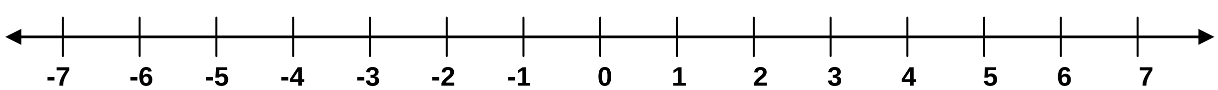
a. –2



b. 4



c. 0



Distance is always positive. The absolute value is always positive. The absolute value of a positive number is that number. The absolute value of a negative number is that number with no negative sign.

Example: $|-3| = 3$ and $|37| = 37$.

Rule to find the absolute value of a number:

1. If the number is positive, the absolute value is that number.
2. If the number is negative, the absolute value is that number without the negative sign.

Now you try!

7. Find the absolute value of the following expressions.

a. $|13|$ **b.** $|-7|$ **c.** $|-400|$ **d.** $|10|$

8. Without doing any math, is $|-(5829-4928)|$ positive or negative? How do you know?

Notes:

∾ End of Lesson 1 ∾

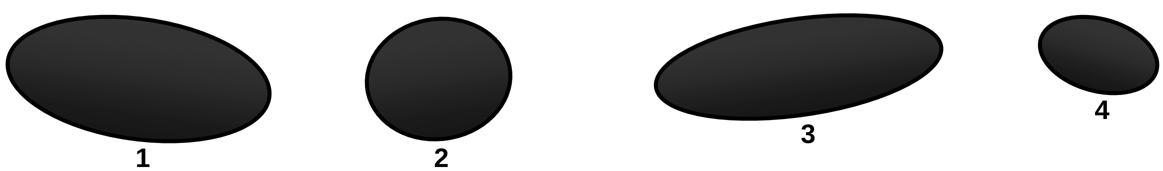
Lesson 2

Addition and Subtraction With Integers

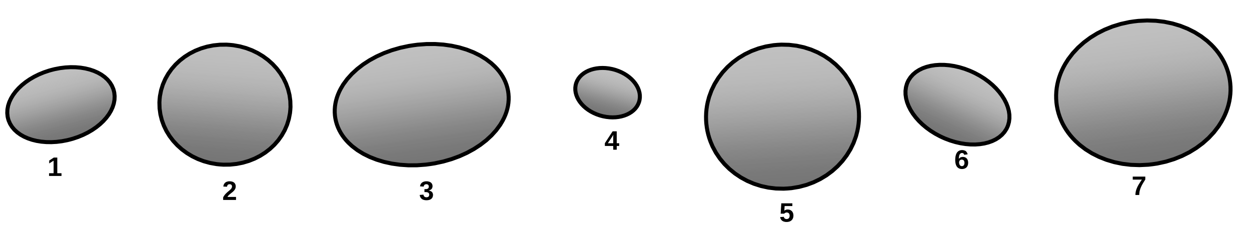
Words to know:

- √ addition
- √ subtraction

Addition is the process used to count the number of objects in two or more groups. Imagine you are walking on a beach, picking up stones. You pick up some dark stones and count them.

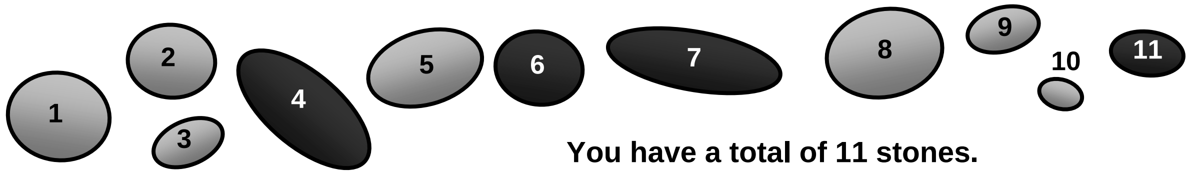


There are four dark stones. You then see how many light colored stones you can find and count those.



You have four dark stones and seven light stones.

How many stones do you have all together? Count them.



You have a total of 11 stones.

You used addition to find the sum of the number of dark stones and light stones. To add 4 and 7, you started at 4, counted 7 more, and ended with 11. You could have also started with 7 and counted 4 more; the answer would have been the same.

- ✓ Addition is used to count the number of objects in two or more groups.
- ✓ The symbol used for addition is “+”.
- ✓ The sum is the number you get when you add two or more numbers together.

For example: 2 + 3 = 5. 9 plus 7 is 16. The sum of 6 and 0 is 6.

There are several ways to show addition in math.

Example: Adding four dark and seven light stones could be written like this:

$$4 + 7 = 11, \qquad \text{or} \qquad \begin{array}{r} 4 \\ +\ 7 \\ \hline 11 \end{array}$$

Both ways of writing the addition problem are correct. The second way is more helpful when adding larger numbers. Here is another example.

Example: Jahmel threw a ball 27 feet. He picked it up and threw it again. The second time it went 25 feet. How many total feet did Jahmel throw the ball? Or, what is the sum of the distances that Jahmel threw the ball?

Solution: Use addition to solve this problem.

Step 1: Line the numbers up on top of each other as shown below.

Step 2: Add the digits farthest to the <u>right</u> first.

$$\begin{array}{r} 27 \\ +\ 25 \\ \hline \end{array}$$

7 + 5 = 12

Step 3: Put the 2 below, and carry the 1 to the next place, as follows.

$$\begin{array}{r} {}^{1} \\ 27 \\ +\ 25 \\ \hline 2 \end{array}$$

Step 4: Now add each digit in the next column.

1 + 2 + 2 = 5

Step 5: Write this sum next to the 2.

$$\begin{array}{r} {}^{1} \\ 27 \\ +\ 25 \\ \hline 52 \end{array}$$

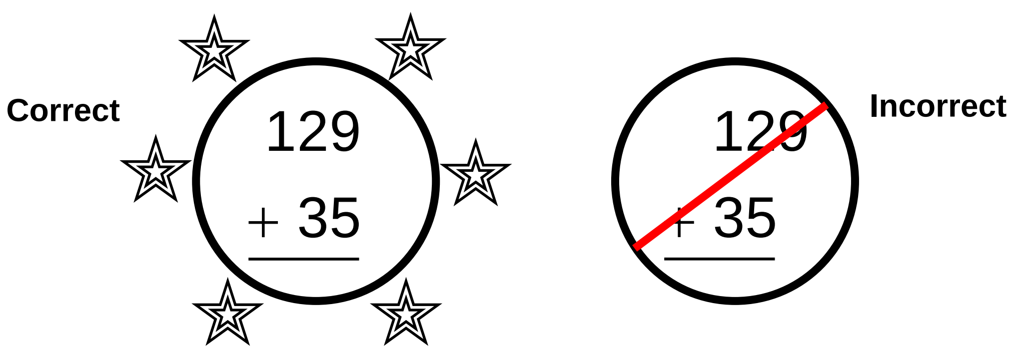
So, Jahmel threw the ball a total of 52 feet.

Rule to add two or more numbers:

1. Write the numbers. Put the digits at the far right of each number on top of one another.
2. Add all the digits. Start with those at the far right.
   a. If the first sum is ten or more, write the digit in the ones place directly under the digits you added.
   b. Write the number in the tens place above the next column to the left.
3. Add the digits in the column to the left of the ones you just added.
   a. If the sum is ten or more, repeat step 2, a. and b.
4. Repeat this process, right to left, until every column of digits has been added.

Example: Tony filled up his gas tank for $35. Later that day, his car broke down. It cost $129 to repair it. How much money in total did Tony spend on his car that day?

Solution: To solve this problem, you must add 35 and 129. When you line up the numbers, make sure the digits farthest to the right are lined up.



Now, follow the steps from before. Work from right to left.

The final sum looks like this.

$$\begin{array}{r} {}^{1} \\ 129 \\ +\ \ 35 \\ \hline 164 \end{array}$$

Tony spent a total of $164 on his car that day.

This method also works for adding more than two numbers.

Now you try!

1. Find the sums

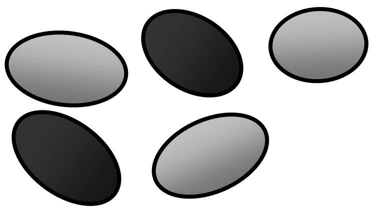
a. 1 + 2 =	**b.** 7 + 2 =	**c**. 11 + 4 =
d. 4 + 6 =	**e.** 7 + 6 =	**f.** 7 + 8 =
g. 14 + 7 =	**h.** 30 + 40 =	**i.** 179 + 5 =

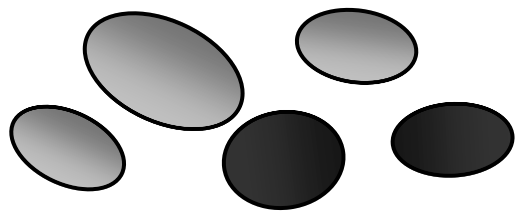
2. Pedro picked 1,247 peaches on Monday and 989 on Tuesday. How many peaches did he pick all together?

3. Find the sum. 124 + 65 + 4

Subtraction

You are back on the beach. This time, you have picked up ten stones. You decide to throw five of them into the water, one at a time. After you throw the first stone, you count down one from ten. That leaves nine stones. Then you throw another stone. Now, you have eight stones left. Then seven, and finally six stones are left in your hand.





stones thrown into the water

The process of taking away is called ***subtraction***.

- ✓ Subtraction is used when things are taken away from a group.
- ✓ The symbol for subtraction is a dash, “–“. It is called a minus sign.
- ✓ The answer to a subtraction problem is called the difference.
- ✓ Three ways to show subtraction:
- ✓ 5 – 3 = 2. 14 minus 2 is 12. The difference between 21 and 7 is 14.

Example: What is the difference between 8 and 5?

Solution: When a question asks you to find the <u>difference</u>, that means subtraction. To find 8 – 5, start from 8 and count backwards 5.

8	7	6	5	4	3
	1	2	3	4	5

So, 8 – 5 = 3

There is another, simpler method to find the difference between two numbers. It uses addition.

Example: Adrian bought groceries that cost $23. He gave the cashier $30. How much change should Adrian receive?

Solution: To find how much change Adrian will receive, you must find the difference between 30 and 23. One way to find this is to ask the question, “23 plus what will give me 30?” In math terms:

$$23 + \_\_\_ = 30$$

Count up from 23 to 30, and keep track of how many that is.

24	25	26	27	28	29	30
(1	**2**	**3**	**4**	**5**	**6**	**7)**

The answer is seven. $23 + \underline{7} = 30$.

Adrian will receive $7 in change.

This method is very helpful for doing subtraction in your head. Many people use their fingers to subtract with this method.

What if you have to subtract large numbers? Will counting and using your fingers work?

Example: Pedro has a bag with 150 candies in it. He gives away 72 pieces of candy on Halloween. How many pieces of candy does he have left?

Solution: Phrases like *give away*, *less, take away*, or *minus* mean subtraction. In this case, the problem says that Pedro <u>gives away</u> 72 pieces from his 150 pieces of candy. That means subtraction: 150 – 72

These two numbers are large. To subtract them, use the same method you used with adding large numbers. Follow the steps below.

$$\begin{array}{r} 150 \\ -\ \ 72 \\ \hline \end{array}$$

Step 1: Write the numbers on top of one another, with the digits on the <u>right</u> perfectly lined up.

Step 2: Starting with the digits on the right, subtract the bottom one from the top one.

$$\begin{array}{r} 150 \\ -\ \ 72 \\ \hline \end{array}$$

How can you find 0 – 2 ?
Two is larger than zero.

You must use the *borrowing method*. Look at the number 150 and think back to place values.

150 has 0 ones, 5 tens, and 1 hundreds.

You can borrow one of the tens from the five tens to perform the subtraction. One ten is worth ten ones. (Think about money. A ten-dollar bill can be exchanged for ten one-dollar bills.) Take 1 ten from the 5 and exchange it for ten ones.

$$\begin{array}{rrr} & 4 & 10 \\ 1 & \cancel{5} & \cancel{0} \\ - & 7 & 2 \\ \hline \end{array}$$

Cross out the 5 and subtract 1 from it. 5 – 1 = 4. Write the 4 above the crossed out 5.

Now cross out the 0, and add 10 to it. 0 + 10 = 10. Now you can subtract the 2 from ten.

Next, look at the 4 and the 7 in the middle column.
Since 7 is larger than 4, you must borrow again,

$$\begin{array}{rrr} & 4 & 10 \\ 1 & \cancel{5} & \cancel{0} \\ - & 7 & 2 \\ \hline & & 8 \end{array}$$

$$\begin{array}{rrr} & 14 & \\ & \cancel{4} & 10 \\ \cancel{1} & \cancel{5} & \cancel{0} \\ - & 7 & 2 \\ \hline & 7 & 8 \end{array}$$

The finished problem will look like this.

Pedro has 78 pieces of candy left.

Rule to subtract two numbers:

1. Write the numbers with the digits at the far right directly on top of one another.
2. Starting with the digits farthest to the right, subtract the bottom from the top.
   - **a.** If the bottom digit is larger than the top digit, borrow one unit from the tens place of the top number. Subtract one from it, and add ten to the ones place of the top number. Then subtract the ones.
3. Subtract the digits in the tens places.
   - **a.** Borrow from the hundreds place, if the bottom digit is larger than the top in the tens place.
4. Repeat this process until every column of digits has been subtracted.

Now you try!

4. Find the differences

a. 3 – 2 =	**b.** 7 – 4 =	**c.** 9 – 3 =
d. 6 – 5 =	**e.** 8 – 1 =	**f.** 11 – 3 =
g. 15 – 7 =	**h.** 13 – 11 =	**i.** 6 – 6 =

5. Solve the next subtraction problems by rewriting them as addition problems (*Ex:* 7 – 4 = __ should be rewritten as 4 + __ = 7)

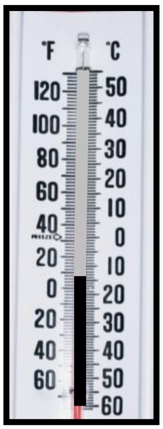
a. 17 – 13 =	**b.** 12 – 9 =	**c.** 56 – 52 =
d. 27 – 22 =	**e.** 13 – 12 =	**f.** 54 – 24 =

6. Tami is saving money for a trip to Hawaii over spring break. The trip will cost $1745. She has saved $1290 for the trip so far. How much more money must she save? _______________

Using Number Lines and Integer Chips

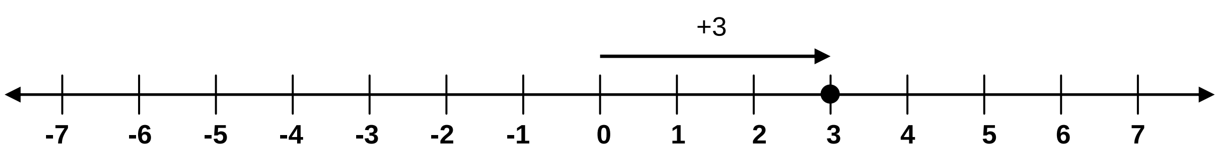
Number lines and integer chips can be used to add and subtract. You may find them useful in real-life situations.

Example: One morning you check the temperature outside. The thermometer has a reading of three degrees. The temperature is supposed to be five degrees colder the next day. What temperature will it be the next day?



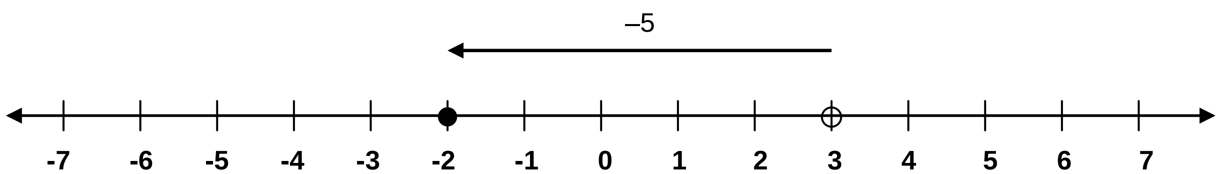
We are now talking about <u>subtracting</u> integers. To find the predicted temperature for the next day, you need to find 3 degrees <u>minus</u> 5 degrees, 3 – 5. To think about this problem, use something you are familiar with. What about a number line?

In this problem, the temperature starts at positive 3 degrees. Plot that on the number line.



When there is no sign in front of a number, it is positive.
3 is a positive number.

The temperature is supposed to go down or decrease by five degrees. In math, “decrease” means subtraction. On a number line, go left for subtraction. Go right for addition. Count five numbers to the left of 3. Put a dot at the new number.



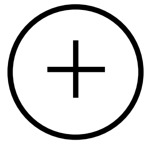
You end up at –2, so 3 – 5 = –2.
The new temperature will be –2 degrees.

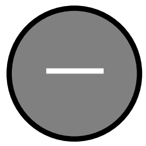
Solving a problem in this way is called the <u>number-line method</u>. Review the steps you followed.

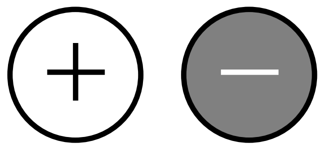
- ✓ Starting at zero, you found the number that began the problem (3).
- ✓ You put a point at its place on the number line.
- ✓ Then, you drew an arrow just above the line, from zero to that number.
- ✓ To subtract, you counted to the left 5 numbers and put a dot at the new number (–2).
- ✓ Then, you drew an arrow, just above the line, from the first number to the new one.

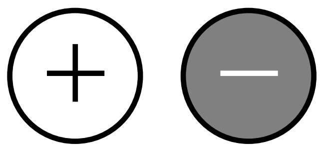
The solution to the problem is the number you ended on: –2.

Another way to solve this problem is the integer chip method. There are two different types of integer chips:

The positive chip  = +1

And the negative chip  = –1

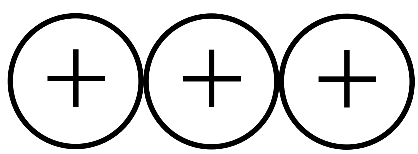
When they are combined,  they equal 0

Since  = –1 + 1 = 0

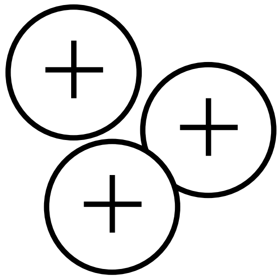
They cancel each other out. To show this, cross them out whenever you see them together.

Now try the temperature problem again using this method.

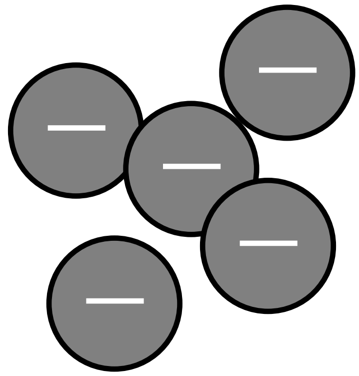
The problem started at positive three degrees. Show this with 3 positive chips.



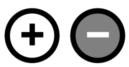
Then, the temperature dropped 5 degrees. Add 5 negative chips to the pile.

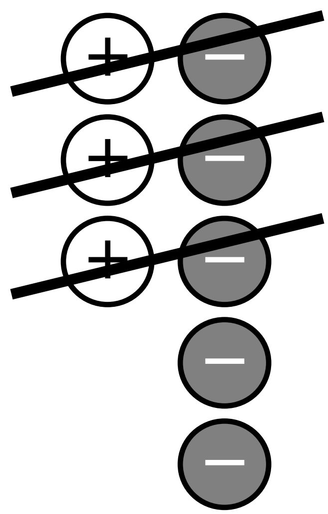


+3



–5

Remember: One positive chip and one negative chip will cancel each other out.
 = 0. Regroup the chips. Make every positive and negative match you can.



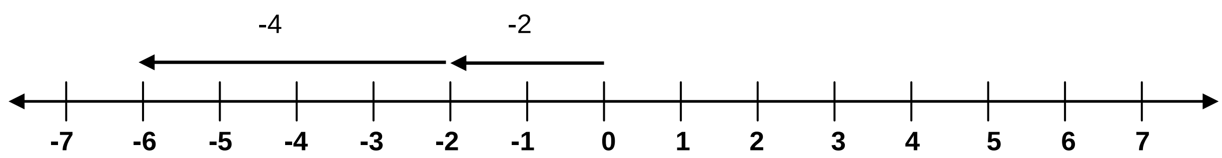
Cancellation!

You can see that 2 negative chips are left.

The temperature tomorrow will be –2 degrees.

Here is another example: What is –2 – 4?

Solution: Use a number line. You have a negative number minus another number. "Minus" means subtraction. You will be moving to the left. Find –2 first. Start at zero on the number line and move two spaces to the left. Now, subtract 4. Move another four spaces to the left.

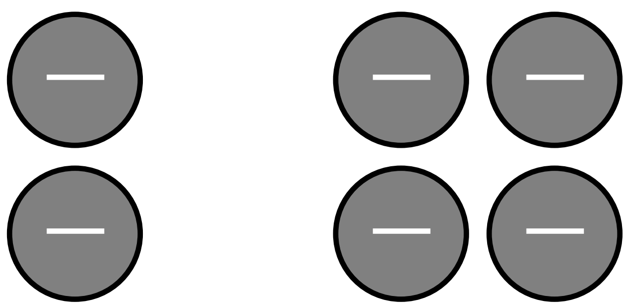


You end up at –6. **So the answer is –2 – 4 = –6.**

You should get the same answer using integer chips. There are actually two ways to solve the problem with chips.

<u>Method 1</u>

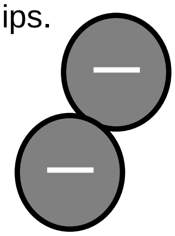
The first way is to think about –2 – 4 as two negative chips plus four more negative chips, or –2 + –4.



You can see that there are 6 negative chips all together. **So, the answer is –6.**

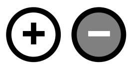
<u>Method 2</u>

The second way to think about –2 – 4 is to start with two negative chips, and take away four positive chips, or –2 – (+4). Here are two negative chips.



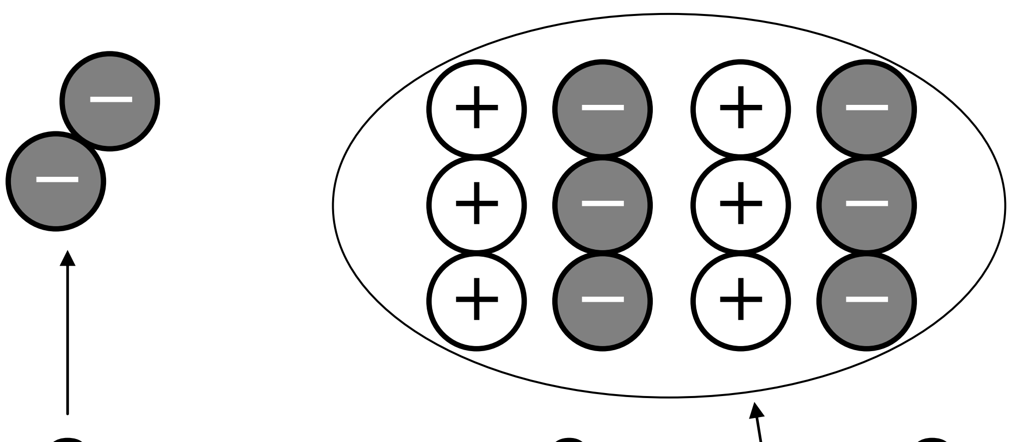
How can you take away 4 positive chips when there are none there?

What if you had a number of positive and negative chips that equaled –2?

Remember:  = 0 and anything plus zero is unchanged.

✓ The identity property of addition says that a number plus zero equals the number. Zero does not change the "identity" of the number. For example, 5 + 0 = 5, or $a + 0 = a$.

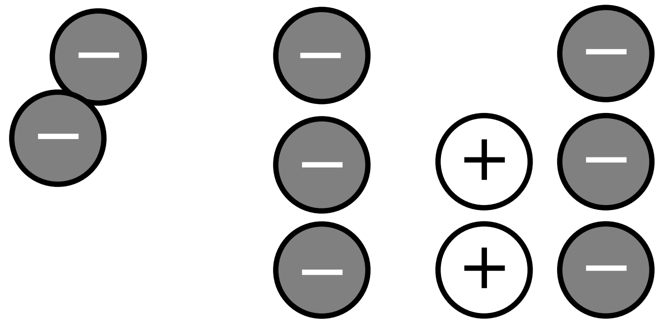
The identity property will let you add pairs of positive and negative chips, without changing value.

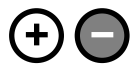


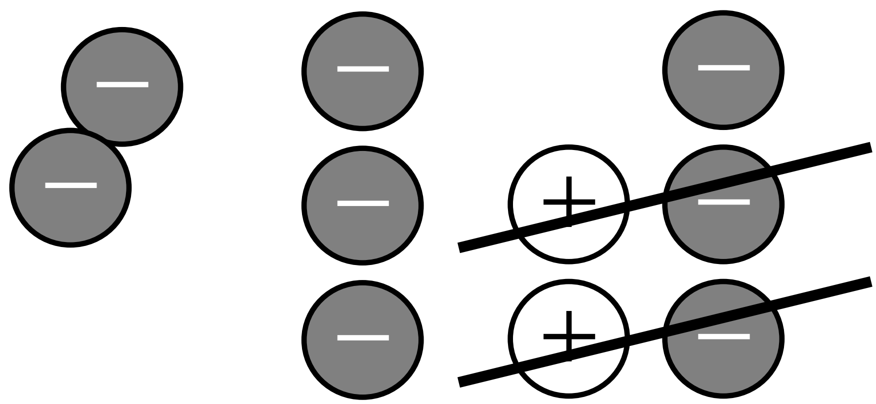
$$-2 \quad + \quad 0 \quad = \quad -2$$

Now you have plenty of chips to use.

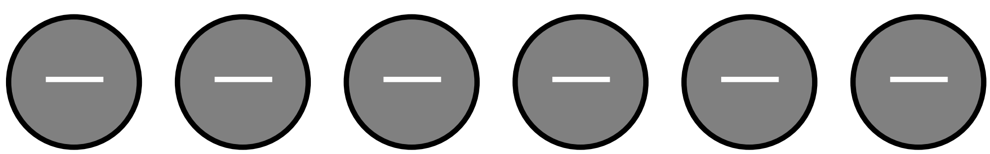
To show –2 – 4 as –2 – (+4), take away four of the positive chips.



Now cancel out groups of 



Count the chips that are left. You can see that there are 6 negative chips.



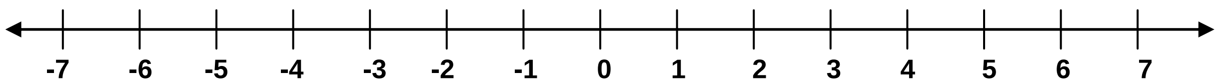
The solution is –6.

The integer chips methods showed that subtraction can be thought of in two ways: as adding negative numbers or as taking away positive numbers!

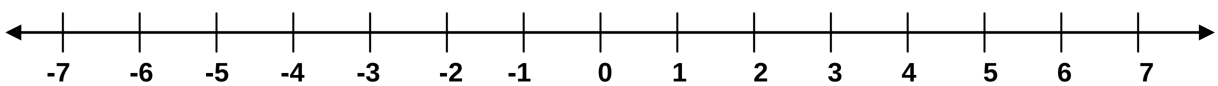
Now you try!

7. Show each of the following using the number line method.

a. –2 + 5



b. 4 – 3



Show these subtraction problems using both integer chip methods (adding negative chips and subtracting positive chips).

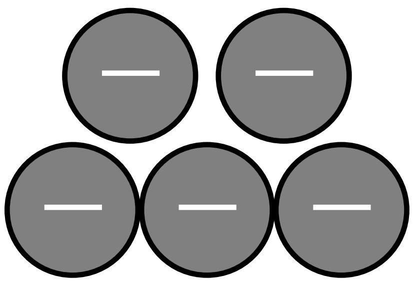
c. 3 – 7

d. 8 – 2

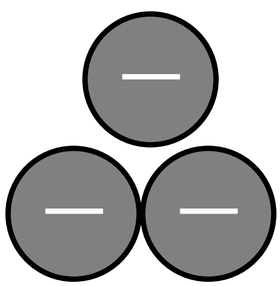
Example: Evaluate –5 – (–2)

Solution: Look at this. You are subtracting a negative number from a negative number! This would be confusing to show with a number line, so let's use our integer chips.

Start with 5 negative chips.



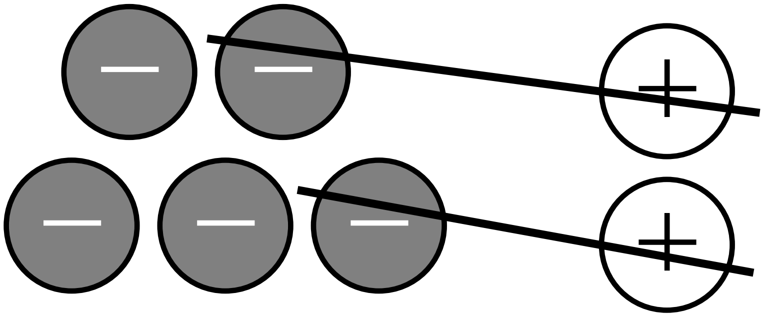
Then, take away 2 negative chips.



You are left with 3 negative chips, or –3.

Parentheses ( ) are sometimes used to make something easier to look at. Here they are used to separate two minus signs.

You just found that –5 – (–2) = –3. What if you evaluate –5 + 2? Using the chips, it will look like the example on the next page:



Cancel groups of plus and minus, because they equal zero.

Again, the answer is –3.

Look at that! Subtracting a negative number is the same as adding a positive number.

$$-5 - (-2) = -5 + 2 = -3$$

Rule to add or subtract integers:

1. Adding a positive integer means add 5 + (+8) = 5 + 8
2. Adding a negative integer means subtract 5 + (–8) = 5 – 8
3. Subtracting a positive integer means subtract 5 – (+8) = 5 – 8
4. Subtracting a negative integer means add 5 – (–8) = 5 + 8

Now you try!

Simplify each expression with only numbers.

9.

- **a.** 3 + 2
- **b.** 6 – 9
- **c.** –4 + (–3)
- **d.** –7 – (–4)

End of Lesson 2

Lesson

3

Multiplication and Division

Words to know:

- √ **multiplication**
- √ **product**
- √ **division**
- √ **quotient**
- √ **dividend**
- √ **divisor**

M***ultiplication*** is repeated addition. The answer to a multiplication problem is called the ***product***. Let's take a look!

Example: You have spent the day at the beach and collected many small stones. Now you want to count them. You decide to make groups of five stones each. You find that you have nine groups of five stones. How many stones is that all together?

5+ 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 45

You have forty-five stones all together. Instead of adding 5 nine times, you could have used multiplication.

- ✓ Multiplication is the same as adding a number many times, or repeated addition. 9 x 5 is the same as adding 5 to itself nine times. There are 9 groups of 5. The symbol we use for multiplication is "×".
- ✓ The answer to a multiplication problem is called the product.
  For example, 3 x 2 = 6. 4 times 3 is 12. The product of 7 and 5 is 35.

Many rules that apply to addition also apply to multiplication. With addition, you can add numbers in any order and still get the same answer.

Example: 3 + 2 = 5 and 2 + 3 = 5.

The same rule is true of multiplication:

Example: 9 x 5 = 45 and 5 x 9 = 45.
Nine groups of five is the same as five groups of nine.

There is a set of steps to follow when you want to multiply larger numbers.

Example: Esmeralda is making cookies at the bakery she works for. She mixes enough dough to fill 16 cookie trays with 12 cookies each. How many cookies will Esmeralda bake all together?

Solution: She is filling 16 trays each with 12 cookies. That means she will have 16 groups of 12. In other words, you must find the <u>product</u> of 16 and 12, or 16 x 12. Start by arranging the numbers vertically, as you did before with addition and subtraction.

$$\begin{array}{r} 16 \\ \underline{\times\ 12} \\ 2 \end{array}$$

Step 1: Multiply the two and the six. 6 x 2 = 12. Put the two from the 12 below the digits column, and carry the one to the tens place.

$$\begin{array}{r} 2 \times 1 = 2 \\ \underline{+\ 1} \\ 3 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 16 \\ \underline{\times\ 12} \\ \mathbf{3}2 \end{array}$$

Step 2: Multiply the two and the one. 2 x 1 = 2. <u>Add</u> this to the 1 that you carried from the first step. 2 + 1 = 3. Write the three below, in the

Step 3: Cross out the 2 and the 1 that you carried. Put a zero beneath the 32 that lines up with the 2.

$$\begin{array}{r} \cancel{1} \\ 16 \\ \underline{\times\ 1\cancel{2}} \\ 32 \\ 0 \end{array}$$

Step 4: Multiply the 1 and 6.
1 x 6 = 6.
Write the 6 under the 3.

Step 5: Multiply the 1 and the other 1. 1 x 1 = 1. Don't do anything with the remainder you crossed out. Write the 1 next to the 6.

$$\begin{array}{r} \cancel{1} \\ 16 \\ \underline{\times\ 1\cancel{2}} \\ 32 \\ \mathbf{16}0 \end{array}$$

$$\begin{array}{r} \cancel{1} \\ 16 \\ \underline{\times\ 1\cancel{2}} \\ 32 \\ \underline{+160} \\ 192 \end{array}$$

Step 6: Finally, add the two products.
32 + 160 = 192

Esmeralda made 192 cookies.

Example: Find the product of 15 x 13**.**

Solution: Here we will illustrate each step in a more condensed way. Study each step from left to right, and notice the changes.

	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$	$\cancel{1}$
15	15	15	15	15
x 13	x 13	x 13	x 13	x 13
	45	45	45	45
		0	150	+ 150
				195

Your multiplication work should look like the column on the far right.

Now you try!

1. Find the products.

a. 3 x 2 =

b. 9 x 7 =

c. 5 x 3 =

d. 7 x 4 =

e. 3 x 9 =

f. 5 x 11 =

g. 2 x 4 =

h. 12 x 5 =

i. 8 x 8 =

2. Find the products. **a.** 27 × 23 **b.** 13 × 13

$$\begin{array}{r} 27 \\ \underline{\times\ 23} \end{array} \qquad \begin{array}{r} 13 \\ \underline{\times\ 13} \end{array}$$

3. Isabel wants to know how much gas she buys in a year. Her car holds 12 gallons of gas. If she fills up her car 24 times all year, how many gallons of gas has she bought that year?

Division

You collected forty-five stones from your day at the beach. Your friend, Alejandro, brings you thirty more. You want to count them in groups of five. How many groups of five stones can you make with 30 stones?

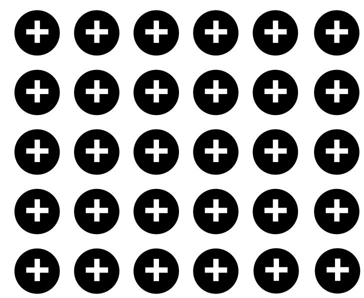
To find out, you must use division. You must find the ***quotient*** of 30 ÷ 5.

Division is the process of separating something into smaller, equally sized groups. It is *repeated subtraction*.

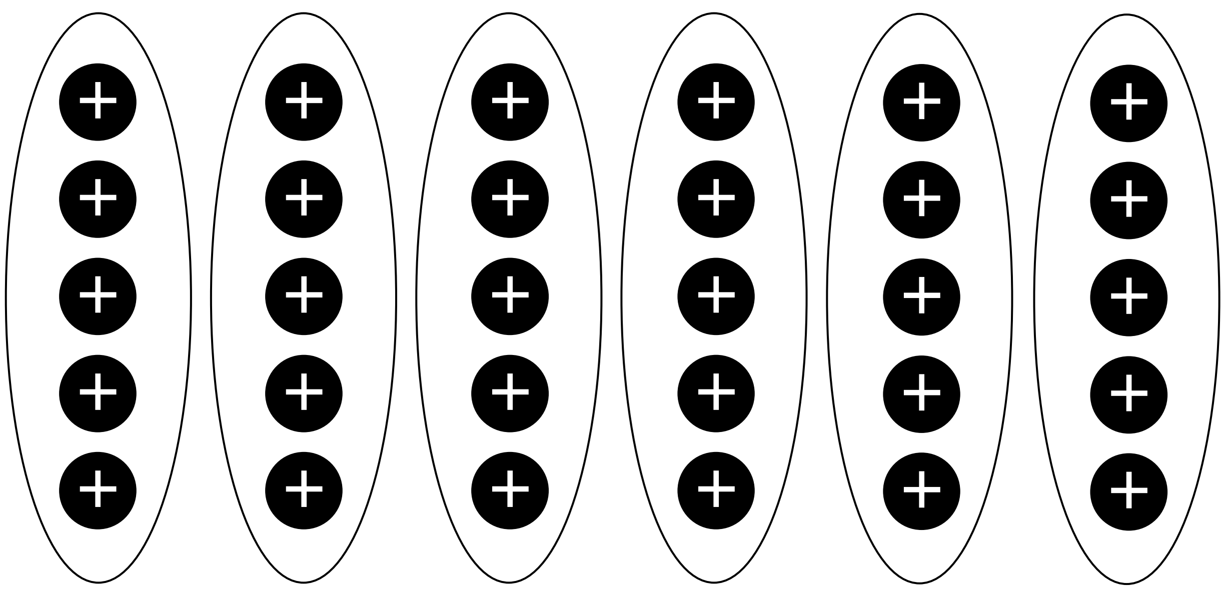
- ✓ The ***quotient*** is the answer to a division problem.
- ✓ The ***dividend*** is the number you are separating into groups (30 above). It is the number you subtract from.
- ✓ The ***divisor*** (5 above) is the number that does the dividing. It can be the size of each group after division. Or, it can be the number of groups a number is divided into.

Look at this example of a division problem: 6 ÷ 2 = 3. Six is the dividend. Three is the quotient. Two is the divisor. It means that there are three groups of two in the number six. It also means that six can be divided into two groups of three.

Now, back to your problem of finding out how many groups of five 30 can be divided into. Use integer chips to represent the stones. Let one positive integer chip represent one stone.



Make groups of 5 positive chips each.



You can make six groups. Six groups of five stones is thirty stones. This sounds like multiplication, doesn't it? Specifically, 6 x 5 = 30. You could have answered this division question by turning it into a multiplication problem.

30 ÷ 5 = ___ is the same as asking 5 x ___ = 30.

Example: Find the quotient of 28 and 4.

Solution: The first thing to understand is that quotient means to divide. You must find the answer to 28 ÷ 4. You know you can change this into a multiplication problem.

4 x ___ = 28

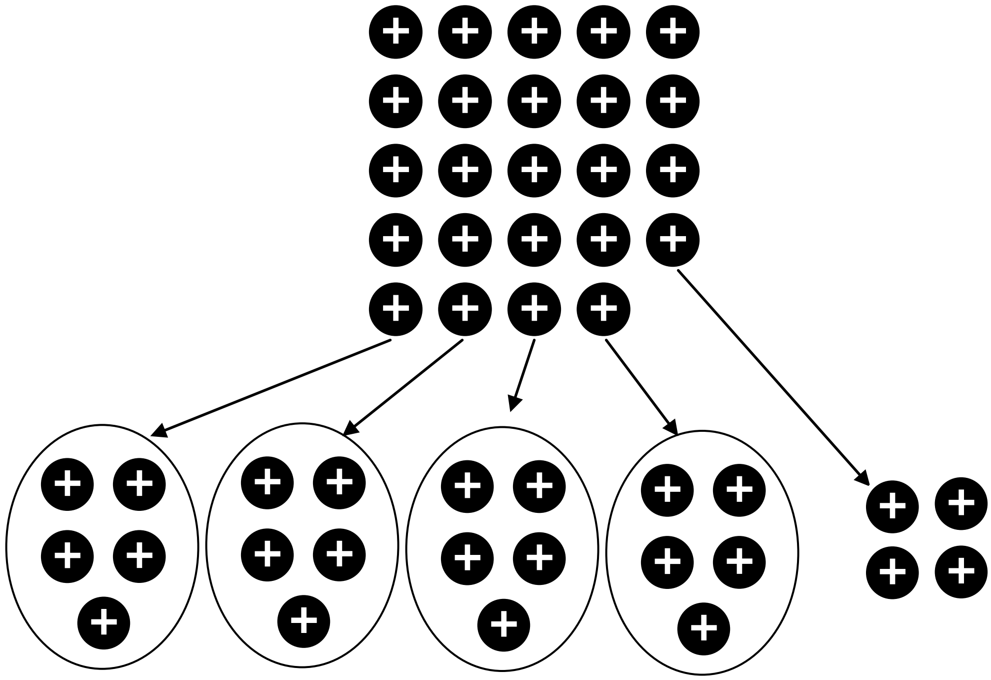
Now the question is easier to answer.

If you do not know this multiplication fact in your head, write out something like this:

$$4 \times 1 = 4$$
$$4 \times 2 = 8$$
$$4 \times 3 = 12$$
$$4 \times 4 = 16$$
$$4 \times 5 = 20$$
$$4 \times 6 = 24$$
$$4 \times 7 = 28$$

You see that 4 x 7 = 28. The answer to the division problem, or quotient, is 7.

Just as you finish solving the division problem, Alejandro brings you more stones. This time, there are 24 stones to divide into groups of 5. With integer chips, 24 ÷ 5 looks like the example on the next page:



There are four perfect groups of 5 stones and 4 stones left over.

In math, this leftover four is called a ***remainder***.

- ✓ The remainder is the amount left over after division. To show a remainder, put an upper case "*R*" after the number of even divisions. Write the remainder number directly after the "R".
  For example, the problem above would be written as:
  24 ÷ 5 = 4 *R4*

FACT

The remainder should always be smaller than the divisor. If it is larger than the divisor, you need to keep dividing until the remainder is less than the divisor.

You learned that multiplication is really repeated addition, or adding many times. Division can be shown as repeated subtraction. You can think of the stones problem, 24 ÷ 5, like this:

“How many times can I subtract 5 from 24 without making it negative?”

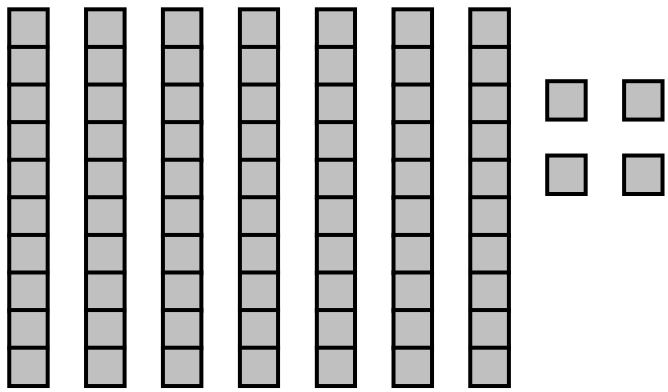
You can subtract 5 from 24 four times, and then have 4 left over. That means your answer is 4 *R*4.

$$\begin{array}{r} 24 \\ \underline{-\ 5} \\ 19 \\ \underline{-\ 5} \\ 14 \\ \underline{-\ 5} \\ 9 \\ \underline{-\ 5} \\ 4 \end{array} \qquad 4\ \ R4$$

Thinking of division as repeated subtraction helps make a rule for dividing larger numbers.

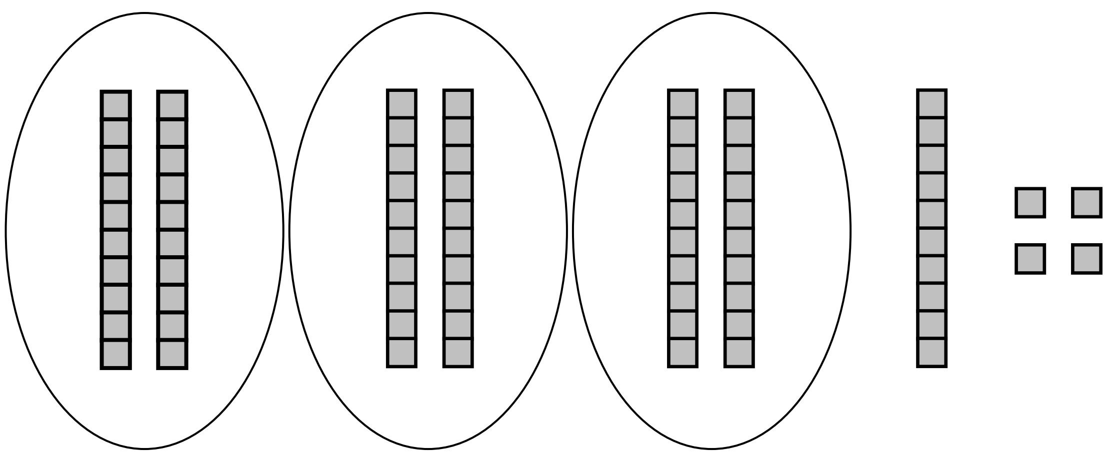
Example: Find 74 ÷ 3.

Solution: In the number 74, 7 is in the tens place, and 4 is in the ones place. You can also say that there are 7 tens and 4 ones. Visually, it looks like this:

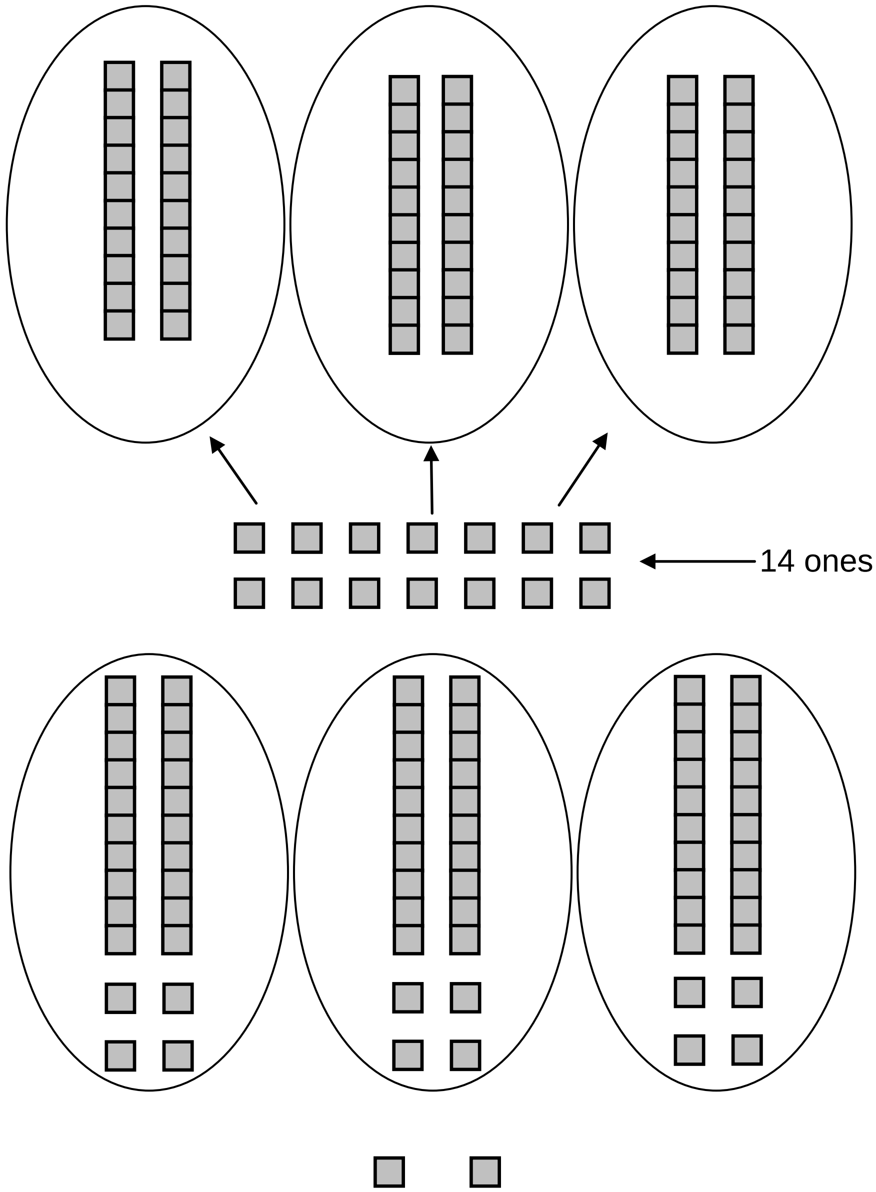


You need to divide 74 into three groups.

First, group the tens.



Two tens evenly fit into each group. One ten and four ones are left over. You cannot divide 10 ones equally among the 3 groups. You must keep it as 10 ones, and add the remaining 4. Now, you have 14 ones to divide into the three groups.



You can see that each group contains 2 tens and 4 ones. There are 2 ones remaining. The 2 ones cannot be divided evenly into the three groups. **So, the answer is 24 *R*2.**

This method can be shown with numbers too.

Instead of writing 74 ÷ 3, you can write $3\overline{)74}$.

It means the exact same thing. Then, just as you did with the blocks, divide the tens place by three.

$$\begin{array}{r} 2 \\ 3\overline{)74} \\ \underline{6} \\ 1 \end{array}$$

Step 1: 7 tens divide into 3 groups of 2 tens

Step 2: 3 groups of 2 tens is 6 tens

Step 3: There is still one ten left to divide, which does not divide evenly into three groups.

At this point in the visual model, you broke the group of ten into 10 ones, and combined it with the 4 ones. This is shown as follows.

$$\begin{array}{r} 24 \\ 3\overline{)74} \\ \underline{-6} \\ 14 \\ \underline{-\ 12} \\ 2 \end{array}$$

Step 4: Combine the 4 ones with the 1 ten to get 14 ones. Now we must divide 14 ones into 3 groups.

Step 5: 14 ones divide into 3 groups of <u>4</u>.

Step 6: 3 groups of 4 ones is 12 ones.

Step 7: Notice there are still 2 ones remaining. This is the remainder.

The final solution, with all the work, will look like this,

$$\begin{array}{r} 24 \quad R\,2 \\ 3\overline{)74} \\ -6 \\ \hline 14 \\ -12 \\ \hline 2 \end{array}$$

***Step 8*:** Write the remainder.

Now you try!

4. Find the quotients by rewriting the division problems as multiplication problems. (For example, to find 10 ÷ 2 = ___, you would write 2 x ___ = 10. Then fill in 2 x $\underline{5}$ = 10.)

a. 12 ÷ 2 = ___ **b.** 16 ÷ 4 = ___ **c.** 50 ÷ 25 = ___

d. 24 ÷ 8 = ___ **e.** 35 ÷ 7 = ___ **f.** 18 ÷ 2 = ___

g. 100 ÷ 4 = ___ **h.** 20 ÷ 5 = ___ **i.** 36 ÷ 12 = ___

Use the step by step method to find the quotients. There may be remainders.

5. $2\overline{)28}$ **6.** $2\overline{)428}$ **7.** $4\overline{)27}$

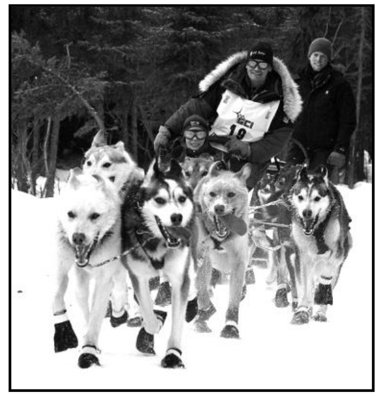
8. $3\overline{)32}$ **9.** $5\overline{)223}$ **10.** $6\overline{)1000}$

11. A school is divided into grades nine, ten, eleven, and twelve. Each grade has the same number of students. If there are 1,424 students in the school, how many students are in grade ten?

Multiplying and Dividing with Negative Numbers

When you multiply and divide negative numbers, you must pay attention to their signs (+ or –).

Example: The Iditarod (I-did-a-rod) is a dog-sled race run in Alaska every year. Temperatures during the race can go far below zero. One year, the temperature at one checkpoint in the race was –35º. From there, the racers traveled up a mountain. At the next checkpoint, the temperature was twice as cold. What is the new temperature at the second checkpoint?



courtesy of Frank Kovalchek

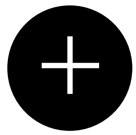
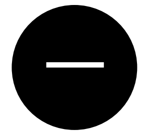
***Solution*:** In this problem, the picture above and the information about the Iditarod are not necessary. Take only what is important. You are told that the first temperature is –35º. The second temperature is twice as cold as –35º. In math terms, you can write this temperature change as

$$\begin{array}{r} -35 \\ \underline{\times\ \ 2} \end{array}$$

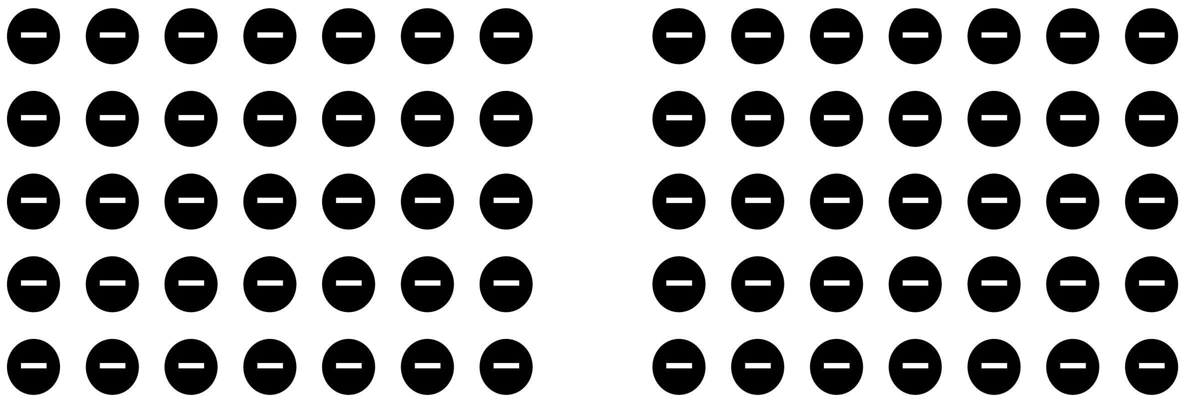
This is the first time you have seen negative numbers with multiplication. Think about the meaning of the statement, –35 x 2. It means "two groups of negative 35." You can show that with integer chips. Think back or look back to Lesson 1.

> ***Think Back***
>
> ***You can multiply numbers in any order you want. It still means the same thing!***
>
> 3 x 2 = 2 x 3 =6

From before,

here is a positive chip  = +1 and a negative chip  = -1.

Now we must show two groups of –35. Here they are.



If you count, there are 70 negative integer chips. They represent –70. That means that –35 x 2 = –70. The temperature at the second checkpoint was –70.

Using integer chips to multiply and divide negative numbers can become difficult. There are rules that make the process easier.

Rule to multiply or divide <u>two</u> integers:

1. Ignore the signs (+ or –) in front of the integers.

2. Multiply or divide as if they are both positive.

3. Write down the product or quotient.

4. Now look back at the signs of the original two numbers.

  a. If they are the same, your answer will be positive (+).

  b. If they are different, your answer will be negative (–).

Example: Simplify 12 ÷ –3

Solution: First set this up as a division problem and solve it ignoring the signs.

$$3\overline{)12} \xrightarrow{\textbf{step 1}} \begin{array}{r}4\\ 3\overline{)12}\end{array} \xrightarrow{\textbf{step 2}} \begin{array}{r}4\\ 3\overline{)12}\\ \underline{-12}\\ 0\end{array}$$

You end up with 4 and no remainder. Now, look back to your original numbers, 12 and –3. The signs here are positive (+) and negative (–). These signs are <u>different</u>, so we know the answer will be negative. **Therefore, 12 ÷ –3 = –4.**

Note that this answer is also the correct solution to –12 ÷ 3.

Example: Find the product. –12 x –9

Solution: First multiply as you would with positive numbers.

$$\begin{array}{r} 12 \\ \underline{\times\ 9} \end{array} \quad \Longrightarrow \quad \begin{array}{r} {}^{1} \\ 12 \\ \underline{\times\ 9} \\ 108 \end{array}$$

Now, look at the signs of the original two numbers, –12 and –9. Both signs are the same, so the answer will be positive.

Therefore, –12 x –9 = 108.

Now you try!

12. Find the products.

a. –2 x –3 ______ **b.** –4 x 2 ______ **c.** 6 x 5 ______ **d.** 8 x –4 ______

e. –9 x –7 ______ **f.** –6 x 4 ______ **g.** 12 x –11 ______ **h.** 8 x 7 ______

13. Find the quotients.

a. –8 ÷ –2 ______ **b.** 14 ÷ –7 ______ **c.** –20 ÷ 10 ______ **d.** 18 ÷ 9 ______

e. –25 ÷ 5 ______ **f.** 24 ÷ –6 ______ **g.** –100 ÷ –10 ______ **h.** –2 ÷ 1 ______

Notes:

End of Lesson 3

Lesson 4

Factors and Multiples

Words to know:

- √ **factors**
- √ **multiples**

F***actors*** are the numbers you multiply together to get another number. Let's take a closer look at factors.

Example: One day, you and your friends decide to play basketball. There are 12 people all together. How many different teams can be made using 12 people?

You could think of this problem using multiplication.

You and your friends are 1 group of 12 people: 1 x 12 = 12.

You could separate into 2 teams of 6 people: 2 x 6 = 12.

Or 3 teams of 4 people: 3 x 4 = 12.

You could also make 4 teams of 3, 6 teams of 2, or even 12 teams of 1. 4 x 3 = 6 x 2 = 12 x 1 = 12

The whole numbers that were used to multiply to 12 are:

1, 2, 3, 4, 6, 12

They are all factors of 12. Notice that 12 is <u>divisible</u> by all of these numbers.

Read the following definition carefully.

✓ When whole numbers, other than zero, are multiplied together, each number is a factor of the product.

Example: 2 and 7 are factors of 14, because 2 x 7 = 14. Similarly, if a whole number divides evenly into a number, the divisor and quotient are factors of that number. 2 and 7 are factors of 14, because 14 ÷ 7 = 2.

In the basketball problem, two of the different ways of grouping were 3 x 4 and 4 x 3.

When you list the factors of a number, count each factor only once. Do not write the same factor twice. Thus, 3 and 4 are listed only once as factors of 12.

Now you try!

1. List all the factors of the following numbers:

a. 24

b. 10

c. 36

Two other factors of 12 are 2 and 6. Notice that 2 has no factors other than 1 and itself, 2. Because of this fact, 2 is defined as a prime number.

✓ A number is ***prime*** if its <u>only</u> factors are 1 and itself. For example, 5 is prime because the only numbers that divide into it evenly are 1 and itself.

The number 6 is not prime. It has more factors than 1 and itself. All the numbers that divide evenly into 6 are: 1, 2, 3, 6

Six has more factors than just 1 and itself. It is called a composite number. It is a composition of many factors.

✓ A ***composite number*** is a whole number greater than 1 that has factors in addition to 1 and itself. For example, 4 is composite because it has the factors 1, 2, and 4.

The composite number 6 can be written as the product of two of its factors, 6 = 2 x 3.

If 12 = 6 x 2,

then 12 = 2 x 3 x 2.

2 x 3 is equal to 6. It can be substituted for 6 in the equation.

You can see that the number 12, written as

12 = 2 x 3 x 2,

has two factors of 2 and one factor of 3.

Written this way, all the factors of 12 are prime numbers.

FACT

The number 1 is neither prime nor composite!

FACT

Every whole number can be written as the product of prime factors! This is a very special property called the Fundamental Theorem of Arithmetic!

One way to factor a number into primes is by using a <u>factor tree</u>.

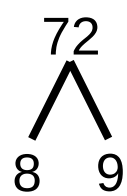
Example: Write 72 as a product of its prime factors.

Solution: You can solve this with the factor tree method.

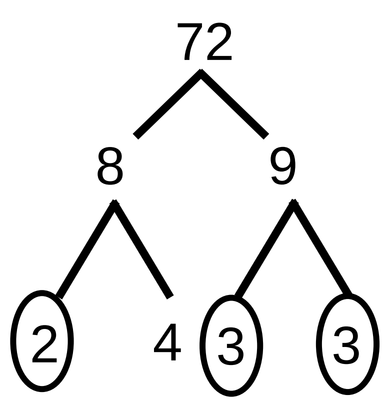
Step 1: Write the number you want to factor.

72

***Step 2*:** Draw two "branches" down from that number. Put two of its factors at the end of the branches. Never use the factor 1.

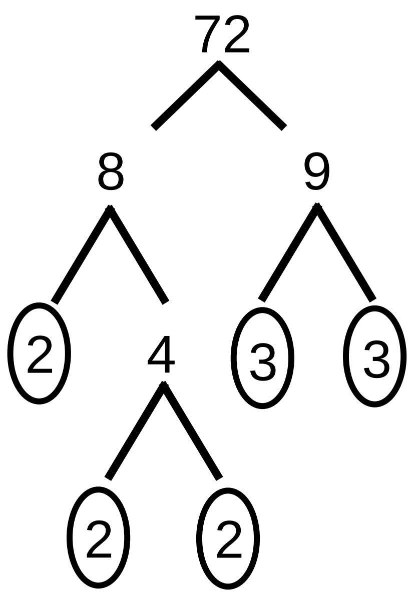


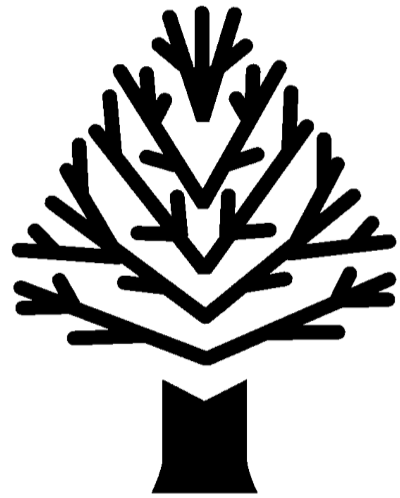
Step 3: Continue to draw branches off each factor, until you have reached a prime number. Circle the prime factors as they occur.



Factor 8 and 9, and circle the prime factors.

Now, factor the 4 and circle its prime factors.

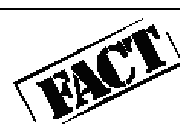




Your factor tree is now complete, but you are not finished yet!

Step 4: Write the answer as a product of prime numbers. The final product is equal to:

72 = 2 x 3 x 3 x 2 x 2 = 2 x 2 x 2 x 3 x 3



The order that you multiply numbers does not matter.

Rule to factor a number:

1. Write the number you wish to factor at the top.
2. Draw two branches below the number. Write the factors of the number at the end of the branches. Do not use 1 or the number as factors unless there are no others.
3. Circle any prime numbers. Continue to factor the composite numbers until all the factors are prime. Circle them.
4. Write the number as a product of its prime factors.

Now you try!

2. Factor each number using a factor tree. Then write the number as a <u>product</u> of prime factors.

a. 64 **b.** 100 **c.** 36

The factor-tree method is very useful for finding the prime factors of a number. You can also use it to find factors that are common to two (or more) numbers.

Comparing the factors of two (or more) numbers:

- ✓ Factors that are not shared are called *unique factors*.
- ✓ Factors that the numbers share are called common factors.
- ✓ The largest factor two (or more) numbers share is their greatest common factor, or their **GCF**. For instance, 2 is the GCF of 4 and 6.

Example: Find the greatest common factor of 90 and 135.

Solution: The steps to solving this problem are:

- list the factors of each number,
- find their common factors, and
- determine which factor is the largest.

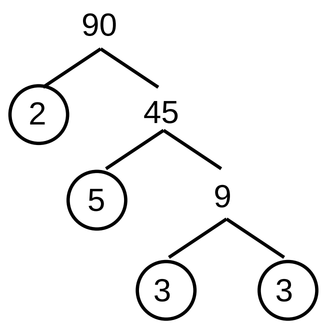
Factors of 90:	1,	2,	3,	5,	9,	10,	18,	30,	45,	90
Factors of 135:		1,	3,	5,	9,	15,	27,	45,	135	

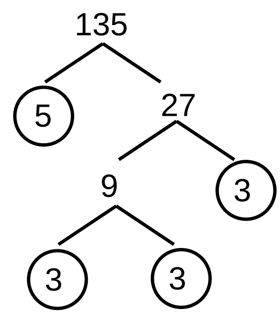
You can see that 45 is the GCF of 90 and 135.

The above method has its problems. It was not efficient to list every factor of 90 and 135. It is also easy to miss factors, and make mistakes. There is an easier way to solve this problem. It uses factor trees and Venn diagrams.

The other method:

Step 1: Factor each number using a factor tree. Rewrite it as a product of prime factors.

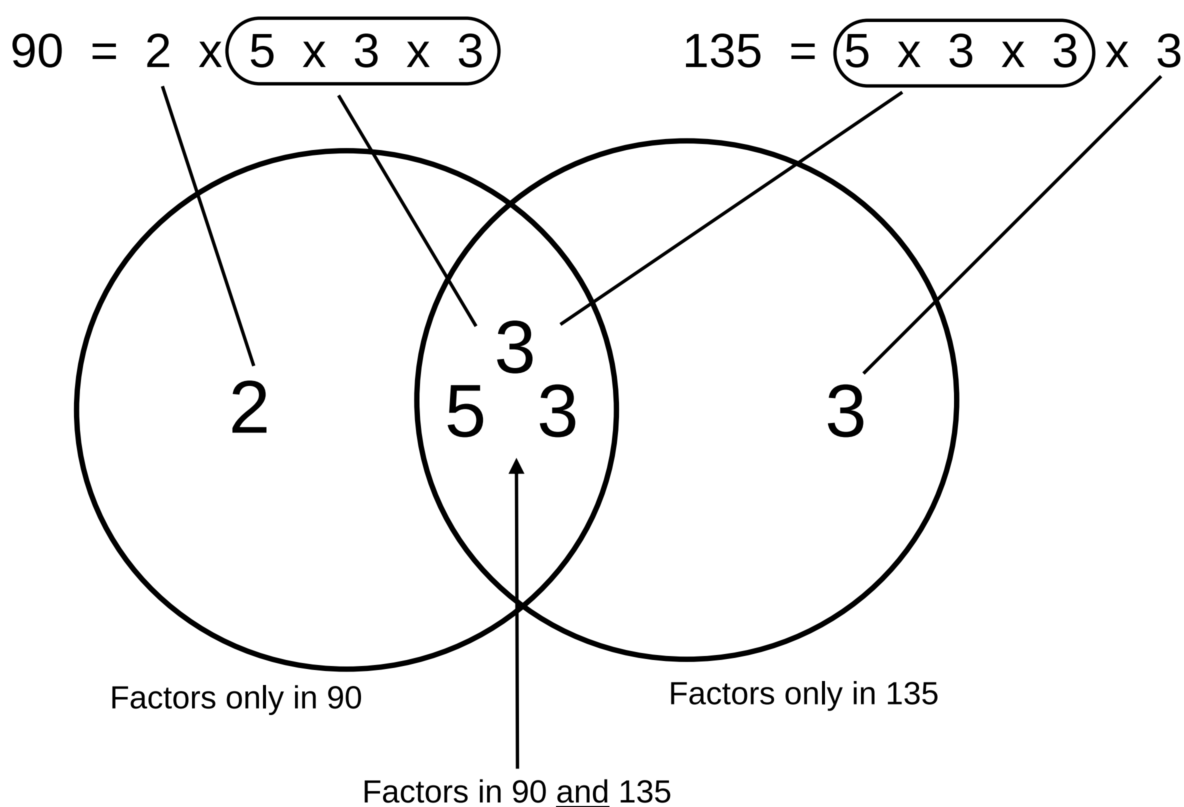




90 = 2 x 5 x 3 x 3

135 = 5 x 3 x 3 x 3

Step 2: Sort using a Venn diagram.



Step 3: The first method showed that the GCF of 90 and 135 is 45.

Look at the common prime factors of 90 and 135.
They are 5, 3, and 3. Notice that $5 \times 3 \times 3 = 45$.

The Venn Diagram method gave you the same answer as the first method. And, it is a good way to avoid forgetting factors!

Rule to find the greatest common factor (GCF):

1. Factor each number and rewrite it as a product of prime factors.

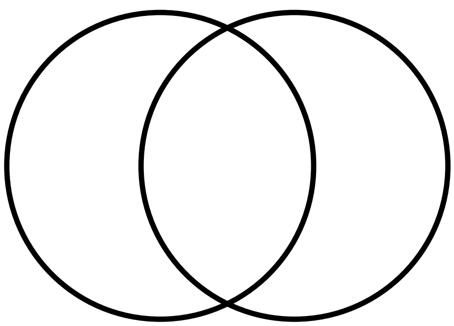
2. Organize the factors of each number using a Venn diagram.

3. Multiply all of the numbers in the center section of the Venn diagram together. This is the GCF.

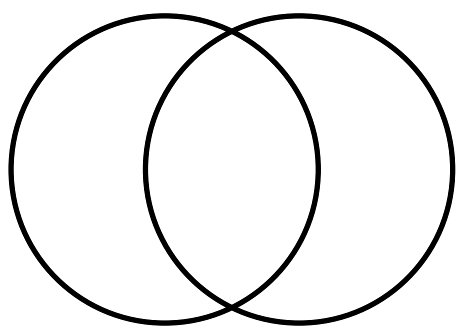
Now you try!

3. Find the greatest common factor for each pair of numbers.

a. 72 and 108



b. 70 and 315



Multiples

You have invited your friends to a barbecue at your house. You need to get hot dogs and rolls for the barbecue. Hot dogs come in packs of 6. Hot dog rolls come in packs of 8. You want the number of hot dogs and rolls to be the same. You will need to buy multiple packs of hot dogs and rolls until you have the same number of each. How can you find out the number of packs of hot dogs and hot dog rolls you need to buy?

This is a multiplication problem. The number of hot dogs you buy will equal the number of packs times 6 hot dogs each. The number of hot dogs you might buy is

6 hot dogs x 1 pack = 6
6 hot dogs x 2 packs = 12
6 hot dogs x 3 packs = 18
6 hot dogs x 4 packs = 24
6 hot dogs x 5 packs = 30
6 hot dogs x 6 packs = 36

The number of rolls you might buy is

8 rolls x 1 pack = 8
8 rolls x 2 packs = 16
8 rolls x 3 packs = 24
8 rolls x 4 packs = 32
8 rolls x 5 packs = 40
8 rolls x 6 packs = 48

Hot dogs come in multiples of 6, and buns come in multiples of 8.

> ✓ A multiple of a number is the product of that number and any whole number besides zero. For example, 20 is a multiple of 4. (4 x 5 = 20)

As you can see, 6 and 8 have some multiples in common.

Multiples of 6:	6,	12,	18,	24,	30,	36,	42,	48..
Multiples of 8:	8,	16,	24,	32,	40,	48,	56,	64…

You can see that both 6 and 8 have the multiples 24 and 48. These are <u>common multiples</u>. In terms of your barbecue, common multiples mean you will have the same number of hot dogs as rolls. The smallest multiple these numbers share is the least common multiple.

> ✓ The smallest multiple two numbers share is called the least common multiple, or LCM.

The least number of hot dogs and rolls you should get is 24. If you buy 4 packs of hot dogs and 3 packs of rolls, you will have 24 of each.

$4 \times 6 = 3 \times 8 = 24$

Example: Find the LCM of 12 and 20.

Solution: List the multiples of each number.

Multiples of 12 are: 12, 24, 48, 60, 72, …

Multiples of 20 are: 20, 40, 60, 80, 100, …

The least common multiple is 60.

FACT ***The LCM of two numbers is the product of those two numbers divided by their GCF.***

Check to see if this fact works.

Factors of 12:	1,	2,	3,	**4**,	6,	12
Factors of 20:	1,	2,	**4**,	5,	10,	20

4 is the GCF. **12 x 20 = 240**

240 ÷ 4 = 60

It works! You can use this fact to check your answer when finding the LCM of two numbers.

Now you try!

4. Find the <u>least common multiple</u> for each pair of numbers.

a. 8 and 16

b. 24 and 84

c. 13 and 17

Notes:

End of Lesson 4

Lesson
5

Fractions

Words to know:

- √ **fraction**
- √ **numerator**
- √ **denominator**
- √ **mixed number**

A ***fraction*** compares parts to a whole. It is the quotient of two numbers: **a** and **b**. A fraction can be written $\frac{a}{b}$ and means $a \div b$. The top number of a fraction is called the ***numerator***. The bottom number of a fraction is called the ***denominator***. A mixed number is the sum of a whole number and a fraction. The + sign is not shown.

It looks like this: $3 + \frac{2}{3} = 3\frac{2}{3}$

Imagine you work in a pizza shop. One night, a family of four comes in and orders a pizza. They ask you to cut it into four equal pieces – one for each member of the family.

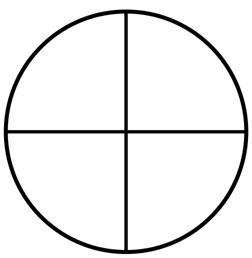
Right after that, a family of five walks in and places the same kind of order. You cut their pizza into five equal slices – one for each member of the family.

Finally, a family of ten walks in, and places the same order! Which pizza will have the biggest slices?

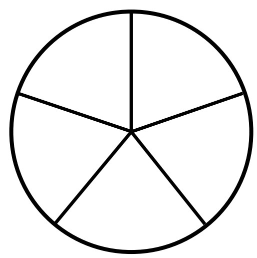
Each pizza ordered was the same size. The first was divided into four equal pieces. The second was divided into five equal pieces. The third was divided into ten equal pieces. Each member of the first family ate more pizza than the members of the other families. Each slice of their pizza was bigger than the slices of the other pizzas. If you cannot see this, don't worry. We can show it with pictures and with math.

Pizza is usually round. Here, a circle represents one whole pizza.

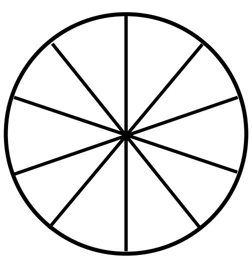
This circle shows the pizza of the family of four. It is divided into four equal parts.



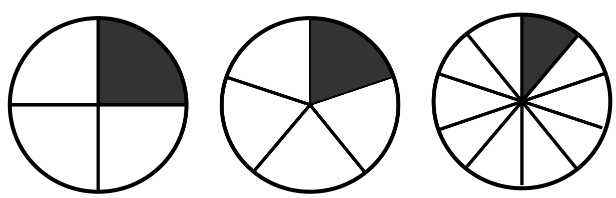
This is the pizza of the family of five. It is divided into five equal pieces.



The pizza of the family of ten is divided into ten equal pieces.



The question asked which family was going to get the biggest slice for each family member. Compare the sizes of the shaded regions of each pizza below.



The pizza cut into four pieces has the largest shaded region. That means that the family of four had the largest slice for each family member.

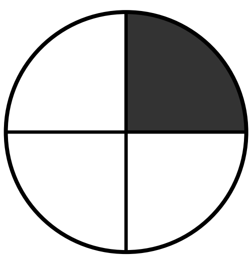
The pizza problem can be shown with math using fractions. Fractions compare the part to the whole.

✓ A fraction is the quotient of two numbers, *a* and *b*. A fraction is written as $\frac{a}{b}$, and it means $a \div b$

Each of the pizzas made for the three different families can be shown using fractions.

Fractions are most often used to represent part of a whole. We would write this as $\frac{\textit{part}}{\textit{whole}}$

The pizza of the family of four is shown below.



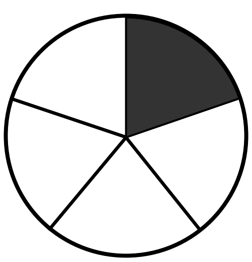
The whole pizza is made of 4 slices.

One slice is 1 part of the whole pizza.

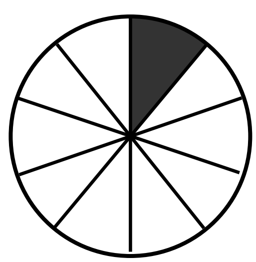
One slice is one of four slices.

This idea can be shown by the fraction, $\frac{1}{4}$

The pizzas of the other families can be shown like this:



$= \frac{1}{5}$



$= \frac{1}{10}$

The shaded area in each drawing is a slice of pizza. It is 1 part of the whole pizza. The whole pizza is made up of the total number of slices. The slice or part is the numerator of a fraction. The total number of parts, or the whole, is the denominator.

- ✓ The top number of a fraction is called the ***numerator***.
- ✓ The bottom number is called the ***denominator***.

$$\frac{\textbf{numerator}}{\textbf{denominator}}$$

The denominator tells you the total number of pieces in the whole. The numerator tells you how many of those pieces you have.

Example: $\frac{1}{2}$ means one out of two pieces, or one-half;

$\frac{1}{3}$ means one out of three pieces, or one-third;

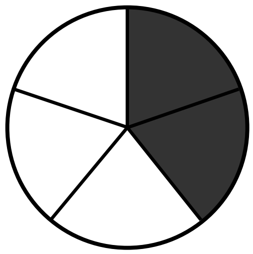
$\frac{1}{4}$ means one out of four pieces, or one-fourth;

$\frac{1}{5}$ means one out of five pieces, or one-fifth, and so on.

The pizza problem focused on 1 individual slice. What happens when you are dealing with multiple slices?

Example: Two people in the family of five decide to save their slices of pizza for later. What fraction of the whole pizza is saved for later?

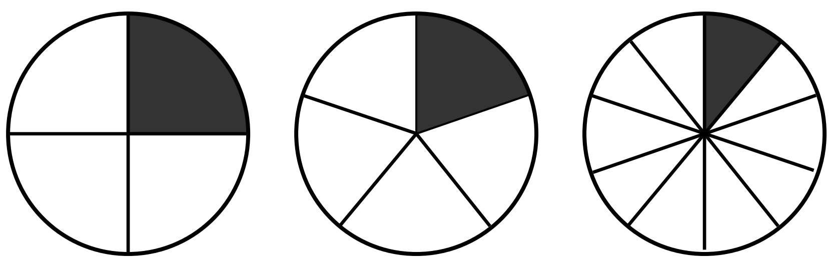
***Solution*:** In this case, there are 5 total slices. Two (2) of them are saved. The denominator will be 5 because there are 5 total slices. The numerator will be 2 because you are focusing on 2 out of 5 slices.



$$= \frac{2}{5}$$

So, $\frac{2}{5}$ of the pizza is saved for later.

Go back to the initial pizza problem.



$\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{10}$

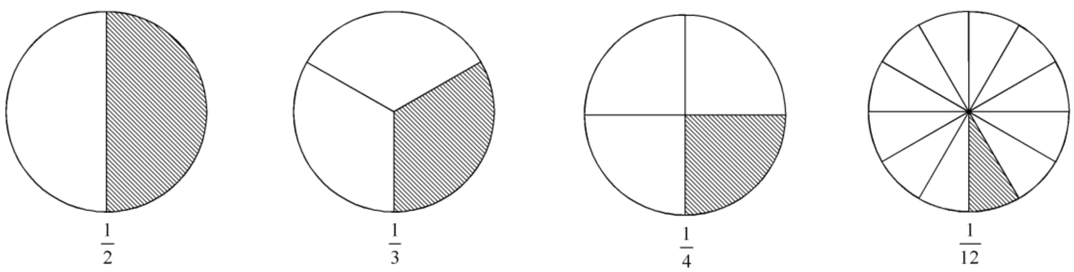
The shaded region of the circle on the left is the largest. The shaded region of the circle on the right is the smallest. You can compare the shaded regions using fractions. It would look like this:

$$\frac{1}{4} > \frac{1}{5} > \frac{1}{10}$$

You can use this method for comparing fractions only if each fraction is part of the same whole object. In this case, identical circles represented one whole pizza.

Example: Compare the size of the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{12}$. Put them in order, from least to greatest.

***Solution*:** We will use pictures to help us answer this question.

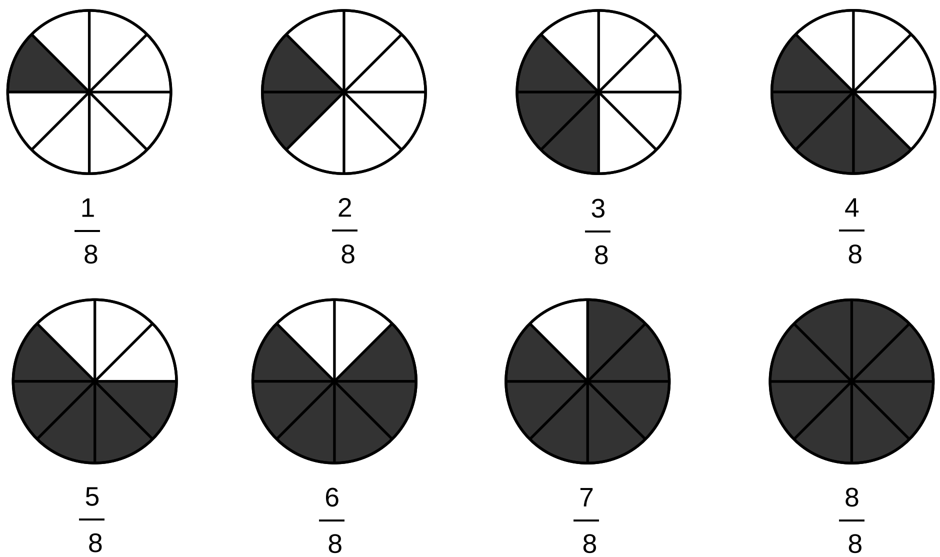


In the pictures above, the same size circle represents one whole. Each circle is divided into equally sized pieces. You can see that more pieces = smaller size. The order of the fractions, from least (smallest) to greatest (largest), is

$$\frac{1}{12} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{2}$$

✓ If a set of fractions has the same numerator, the fraction with the smallest denominator is largest in value.

What if fractions have the same denominators, but <u>different numerators</u>? Look at the example below. What do you notice?



As the numerator grows, so does the value of the fraction. Why is this? Remember: The numerator tells you how many pieces of the whole the fraction has. The fraction $\frac{1}{5}$ tells you have one piece of a whole with five pieces, or one-fifth. How does this compare to the fraction $\frac{2}{5}$?

You still have fifths, but now there are two of them.

$\frac{2}{5}$ must be bigger than $\frac{1}{5}$.

Try some fraction problems on your own.

1. Circle which fraction is larger.

a. $\frac{1}{11}$ or $\frac{1}{9}$ **b.** $\frac{6}{17}$ or $\frac{6}{15}$ **c.** $\frac{13}{19}$ or $\frac{11}{19}$

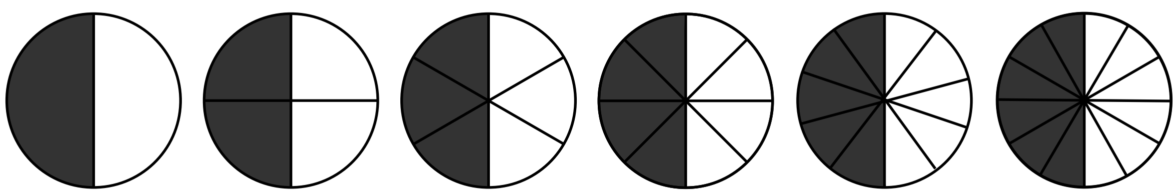
Comparing negative and positive fractions works the same way as comparing negative and positive integers.

Example: Think about $-\frac{2}{3}$ and $\frac{1}{3}$. The first fraction might seem bigger, because its numerator is 2. But, $-\frac{2}{3}$ is a negative number and less than 0. One-third ($\frac{1}{3}$) is a positive number and greater than 0.

$$-\frac{2}{3} < \frac{1}{3}$$

Equivalent Fractions

Sometimes, fractions that look different may be equal in value. Look at the picture form of these fractions.



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$ $\frac{5}{10}$ $\frac{6}{12}$

You can see that the shaded portions of the circles all equal each other. They are all one half of the circle. The only difference is the number of equal pieces each circle has.

You can use algebra to show why the fractions are all equivalent – have equal value. Once again, the equal forms of $\frac{1}{2}$ are $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, and, $\frac{6}{12}$ ...

Do you see a pattern with the numbers? Let's rewrite the fractions to show more clearly how they are equal.

$$\frac{1}{2}=\frac{1\times1}{2\times1}=\frac{1}{2}\left(\frac{1}{1}\right)$$

$$\frac{2}{4}=\frac{1\times2}{2\times2}=\frac{1}{2}\left(\frac{2}{2}\right)$$

$$\frac{3}{6}=\frac{1\times3}{2\times3}=\frac{1}{2}\left(\frac{3}{3}\right)$$

$$\frac{4}{8}=\frac{1\times4}{2\times4}=\frac{1}{2}\left(\frac{4}{4}\right)$$

$$\frac{5}{10}=\frac{1\times5}{2\times5}=\frac{1}{2}\left(\frac{5}{5}\right)$$

$$\frac{6}{12}=\frac{1\times6}{2\times6}=\frac{1}{2}\left(\frac{6}{6}\right)$$

FACT

Any number divided by itself equals one. For example $\frac{5}{5}$ *= 1 and* $\frac{z}{z}=1$

Each equal form of $\mathbf{\frac{1}{2}}$ is simply $\mathbf{\frac{1}{2}}$ multiplied by something equal to one!

Each fraction above can be written as $\frac{1}{2}\times1$. The identity property says that $\frac{1}{2}\times1=\frac{1}{2}$. Thus, every fraction above equals $\frac{1}{2}$.

FACT

The <u>identity property of multiplication</u> states that any number multiplied by 1 is that number. It allows you to multiply any fraction by 1 without changing its value!

You can use the identity property to find equivalent fractions for *any* fraction.

Example: Write two fractions that are equivalent to $\frac{1}{3}$.

Solution: If you multiply both the numerator and denominator by 2, you will see that

$$\frac{1}{3} = \frac{1\times 2}{3\times 2} = \frac{2}{6}$$

Or you can multiply both the numerator and denominator by 3 and get

$$\frac{1}{3} = \frac{1\times 3}{3\times 3} = \frac{3}{9}$$

Thus, $\frac{2}{6}$ and $\frac{3}{9}$ are equivalent to $\frac{1}{3}$.

Now you try!

2. Write two fractions that are equivalent to:

a. $\frac{3}{5}$

b. $\frac{2}{3}$

3. Complete the equivalent fraction.

a. $\frac{3}{5} = \frac{12}{\square}$

b. $\frac{16}{24} = \frac{\square}{12}$

Equivalent fractions can be useful. They can help you rewrite fractions using smaller numbers.

Example: $\frac{30}{45}$ is a fraction with large numbers. You can simplify it using equivalent fractions and common factors.

✓ A fraction is in simplest form or lowest terms, if the numerator and denominator share no common factors. Thus, the fraction has no equivalent forms with smaller numbers.

Simplest form and lowest terms mean the same thing. To "simplify" means to put something into simplest form or lowest terms.

Example: For instance, $\frac{6}{9}$ is not in lowest terms. The numerator and denominator share a common factor of 3.

$\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$ Now it is in lowest terms.

Rule to put a fraction in lowest terms:

1. Find the Greatest Common Factor of the numerator and denominator.

2. Divide the numerator and denominator by that factor.

Remember: If the numerator and denominator of a fraction share no factors, it is in its simplest form. If a fraction is in simplest form, the GCF of its numerator and denominator is 1.

Example: Write $\frac{30}{45}$ in lowest terms.

Solution

Steps 1 & 2: Write the factors of the numerator and denominator. Underline, circle, or highlight the factors they have in common.

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30
Factors of 45: 1, 3, 5, 9, 15, 45

Step 3: You can see that fifteen is the greatest common factor.

Step 4: Divide the numerator and denominator by the GCF.

$$\frac{30}{45} = \frac{30 \div 15}{45 \div 15} = \frac{2}{3}$$

Now you try!

4. Write the following fractions in simplest form.

a. $\frac{4}{12}$ **b.** $\frac{6}{15}$ **c.** $\frac{4}{5}$

You can use the cross product method to check if fractions are equivalent.

Cross Product Method

- ✓ Multiply the numerator of the first fraction by the denominator of the second fraction.
- ✓ Then, multiply the denominator of the first fraction by the numerator of the second fraction.
- ✓ If these two products are equal, the fractions are equivalent.

If $\frac{1}{2} = \frac{2}{4}$, then $(1 \times 4) = (2 \times 2)$. If $\frac{a}{b} = \frac{c}{d}$, then $(a \times d) = (b \times c)$.

It is called the cross product method, because you multiply <u>across</u> the equals sign as shown below.

Example: Show that $\frac{1}{4} = \frac{3}{12}$ using the cross product method.

***Solution*:** Write the fraction equation.

Multiply 1 x 12 and then, 4 x 3, as shown.

$$\frac{1}{4} = \frac{3}{12}$$

$$(1 \times 12) = (3 \times 4)$$

$$12 = 12 \;\checkmark$$

The cross products are equal.

The fractions are equivalent.

Now you try!

5. Which fraction is *<u>not</u>* equivalent to $\frac{2}{3}$? (Circle the correct answer.)

a. $\frac{2}{4}$ **b.** $\frac{6}{9}$ **c.** $\frac{4}{6}$ **d.** $\frac{20}{30}$

6. Decide whether each fraction is in simplest form. Simplify any fraction that is not.

a. $\frac{8}{16}$ **b.** $\frac{12}{18}$ **c.** $\frac{9}{10}$ **d.** $\frac{13}{64}$

Mixed Numbers

Fractions represent parts of whole numbers. They are often combined with whole numbers in everyday life. A whole number plus a fraction is called a mixed number.

✓ A mixed number is the sum of a whole number and a fraction. When written, the addition sign is still there, but it is hiding.

$$3+\frac{2}{3} = 3\frac{2}{3}\text{, and } A+\frac{b}{c} = A\frac{b}{c}.$$

Example: A grape picker picks enough grapes to fill three large barrels and $\frac{2}{3}$ of a fourth barrel. This can be shown by the model below.

$$1+1+1+\frac{2}{3}$$

You can see that $1+1+1+\frac{2}{3}=3+\frac{2}{3}$.

$$3+\frac{2}{3} = 3\frac{2}{3}.$$

Mixed numbers are not whole numbers. They are between two whole numbers. In the example above, the grape picker picked three barrels of grapes plus part of a fourth barrel. So, the mixed number $3\frac{2}{3}$ is between the whole numbers 3 and 4.

There is a specific way to say mixed numbers. You say $3\frac{2}{3}$ as "three and two-thirds".

The mixed number, $5\frac{3}{4}$, is said as, "five and three-fourths".

Notice: the word "and" comes between the whole number and the fraction.

Mixed numbers can be made into fractions.

Use the definition and work backwards.

$$3\frac{2}{3} = 3 + \frac{2}{3}$$

Now, break up the 3.

$$= 1 + 1 + 1 + \frac{2}{3}$$

Substitute the equivalent fraction for each 1 – three-thirds.

$$= \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{2}{3}$$

$$= \frac{9}{3} + \frac{2}{3}$$

You can add the numerators because the denominators are the same – 3.

$$= \frac{11}{3}$$

✓ If the numerator of a fraction is less than its denominator, it is called a *proper fraction*. If the numerator of a fraction is greater than or equal to its denominator, it is called an *improper fraction*.

For instance, $\frac{13}{3}$ is an <u>improper</u> fraction because the top, 13, is bigger than the bottom, 3. The fraction $\frac{7}{18}$ is a <u>proper</u> fraction, since 7 is smaller than 18.

Mixed numbers and whole numbers can always be shown as improper fractions.

When $3\frac{2}{3}$ was changed into an improper fraction, you saw that $3\frac{2}{3}$ really meant "three wholes and two thirds". Then, three wholes was changed to the equivalent nine-thirds. Finally, nine-thirds was combined with two-thirds to get the answer, $3\frac{2}{3} = \frac{11}{3}$.

This method can be used to convert any mixed number into an improper fraction.

Rule to convert a mixed number to an improper fraction:

1. Multiply the whole number part by the denominator of the fraction.
2. Add this product to the numerator of the fraction.
3. Put the fraction into simplest form.

$$5\frac{3}{4} = 5\frac{3}{4} = \frac{(5 \times 4) + 3}{4} = \frac{23}{4}$$

Now you try!

7. Write the mixed numbers as improper fractions.

a. $1\frac{1}{3}$ **b.** $2\frac{7}{8}$ **c.** $3\frac{3}{4}$ **d.** $5\frac{3}{5}$

What if you have an improper fraction and want to make it into a mixed number?

Example: Write $\frac{9}{4}$ as a mixed number.

Solution: In this example, you are working with fourths. You already know that there are four fourths in every whole,

because $\frac{4}{4}=1$.

The numerator of the fraction tells you how many pieces you have. In this case there are nine pieces of fourths. What you need to know is how many groups of four are in nine. To find that, divide nine by four.

> *Think Back*
>
> *Fractions mean division.*
>
> $\frac{9}{4}=9\div 4$

$$\begin{array}{r} 2 \\ 4\overline{)9} \\ -\underline{8} \\ 1 \end{array}$$

There are two wholes and one remaining fourth. The answer is

$$\frac{9}{4}=2\frac{1}{4}.$$

You can use this method to convert any improper fraction to a mixed number.

Rule to convert an improper fraction to a mixed number:

1. Divide the fraction's numerator by its denominator.
2. The number of times it divides evenly is the "whole" part of the mixed number.
3. To the right of that, write a fraction. The numerator will be the <u>remainder</u> found in step 2. The denominator will be the same as that of the original fraction.

$$\frac{9}{2}$$
$$=9\div 2$$
$$=4R1$$
$$=4\frac{1}{2}$$

Example: Write $\frac{14}{3}$ as a mixed number.

Solution

First write $\frac{14}{3}$ as a division problem. $3\overline{)14}$

Find the quotient with the remainder.

$$\begin{array}{r} 4\ R\ 2 \\ 3\overline{)14} \\ -12 \\ \hline 2 \end{array}$$

The 4 stays on the left as the whole number.

The 2 becomes the numerator of the fraction.

The 3 becomes the denominator of the fraction.

$$4\frac{2}{3}$$

8. Write the improper fractions as mixed numbers in simplest form.

a. $\frac{5}{3}$

b. $\frac{21}{8}$

c. $\frac{5}{4}$

d. $\frac{11}{5}$

End of Lesson 5

Lesson 6

Decimals

Words to know:

- √ **decimal**
- √ **terminating decimal**
- √ **repeating decimal**
- √ **rounding**

A ***decimal*** can represent a whole number or the fractional part of a number. Let's take a closer look at decimals.

The price of bananas at your local grocery store is ten bananas for one dollar. You can show the price using a fraction. One banana costs

$$1 \div 10 = \frac{1}{10} \text{ of a dollar.}$$

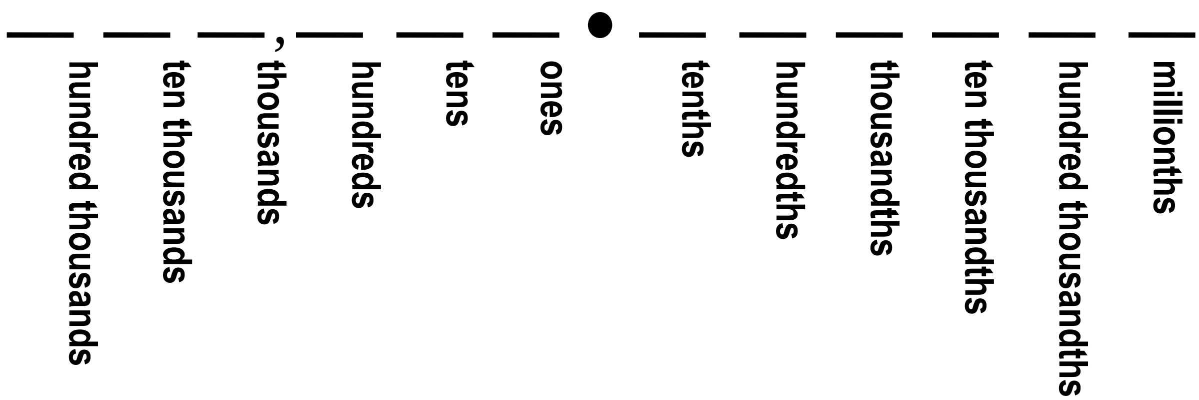
You know that one tenth of one dollar is a dime, or $0.10. So,

$$\frac{1}{10} = .10$$

Here is another way to represent fractions. 0.10 is an example of a decimal.

✓ A decimal is a number that can represent a whole part or a fractional part. A period (.) separates the whole part from the fractional part. It is known as a decimal point. For example, 3.5 is a decimal and so is 0.72.

A good way to understand decimals is to think of them as money. The first number to the right of a decimal point is in the tenths place. With money, one dime is \$0.10, or one-tenth of a dollar. The second number to the right of a decimal point is in the hundredths place. With money, this number tells you the number of pennies you have. One penny is 1 one-hundredth ($\frac{1}{100}$) of a dollar. Or, 100 pennies equal one dollar. The diagram below shows whole number and decimal places. Numbers to the left of the decimal point are whole numbers. Numbers to the right of the decimal point are fractions.



Example: Write the place value of each digit in the number **.**123450

Solution: 1 is in the tenths place. 2 is in the hundredths place. 3 is in the thousandths place. 4 is in the ten-thousandths place. 5 is in the hundred-thousandths place. Zero is in the millionths place. You can also say that there is 1 tenth, 2 hundredths, 3 thousandths, 4 ten-thousandths, 5 hundred-thousandths, and zero millionths.

Now you try!

1. Write each digit in the correct place value in the chart below. Then write the place value of the digit that is farthest to the right.

a. ______________________________

b. ______________________________

c. ______________________________

d. ______________________________

e. ______________________________

a.			•					
b.			•					
c.			•					
d.			•					
e.			•					
	Tens	**Ones**	•	**Tenths**	**Hundredths**	**Thousandths**	**Ten-Thousandths**	**Hundred-Thousandths**

How do you write and say the entire decimal?

Example: Write out 3.413 using words.

Solution: This is a mixture of whole numbers and a fractional part. By counting the number of decimal places, you can see that this number goes to the thousandths place. Say it like this:

3.413 = three and four hundred thirteen thousandths.

- ✓ The whole number is said the way it usually is said.
- ✓ "And" means that there is a decimal point there.
- ✓ The numbers after "and" are read as a fraction. The place value of the digit farthest right is the denominator. In the above example, it is thousandths.

Decimals are read the way mixed numbers are read. Without much work, decimals can be written as mixed numbers.

$$3.413 = 3\frac{413}{1000}$$

Once again, the number is read as three and four hundred thirteen thousandths.

Using mixed numbers, you can change decimals into improper fractions, too!

$$3\frac{413}{1000} = \frac{3000}{1000} + \frac{413}{1000} = \frac{3413}{1000}, \quad \text{so} \quad 3\frac{413}{1000} = \frac{3413}{1000}.$$

Rule to write a decimal with words:

1. Write the number to the left of the decimal as you would any whole
2. number.
3. In place of the decimal point, write the word “and”.
4. Write the number to the right of the decimal point, as you would any whole number.
5. At the end, write the final digit’s place value. It should end in “ths.” (ten<u>ths</u>, hundred<u>ths</u>, thousand<u>ths</u>, …)

Rule to write a decimal as a mixed number:

1. Rewrite all the digits to the left of the decimal point. This is the whole
2. number part.
3. Write all the digits to the right of the decimal point as the numerator of a fraction.
4. For the denominator, write the place value of the right-most digit. (10, 100, 1000, 10000, 100000, …)

For example, $17.927 = 17\frac{927}{1000}$

Now you try!

2. Write each decimal using words, then as a mixed number, then as a fraction in lowest terms.

a. 2.6

Words: ____

Mixed Number:

Fraction:

b. .43

Words: ____

Mixed Number:

Fraction:

c. 1.6524

Words: ____

Mixed Number:

Fraction:

Consider two whole numbers, 340 and 00340. Believe it or not, 340 = 00340. The number 00340 looks strange. Numbers are not usually written this way. The first two zeros before the 3 have no meaning. However, the zero after the 4 is needed. If you drop the zero at the end of 340, the value of the number changes.

Similar things can be done with decimals. The following decimals are all equal.

$$\begin{aligned} &0.43 \\ &= 0.430 \\ &= 0.4300 \\ &= 0.43000 \\ &= 0.430000 \\ &= 0.43000000000000000000 \end{aligned}$$

They are the same because the place value of the 4 and 3 never change.

Any number of zeros may be added ***at the end of a decimal*** *without changing the value of the decimal.*

Now you try!

2. True/False. Decide whether each equation is true or false. Put "T" or "F" on the lines provided.

a. ____ 07 = 7 **b.** ____ 4 = 40

c. ____ 00030 = 00300 **d.** ____ 3.4 = 03.4

e. ____ 8.42300 = 8.423 **f.** ____ 900.163200 = 0900.1632

Knowing this helps us put decimals in order.

Example: Which is larger, .2 or .19?

Solution: You might think that .19 is larger than .2, since 19 > 2. But think about this first.

You know that .2 = .20
In terms of money, you also know that \$0.20 is more money than \$0.19.
Therefore, .2 > .19 .

What about the next two decimals?

Example: Which is larger, 0.2 or .1999999999999999?

Solution: Line up the two numbers according to their place values.

.2
.1999999999999999

Notice that the top number has 2 tenths, and the bottom has only 1 tenth plus something that is less than one tenth, so:

$.2 > .1999999999999999$

Decimals are helpful when comparing the size of two numbers. That is why they are used for money instead of fractions. What you have seen here will help you use the next method of comparing the size of two decimals.

Rule to compare the size of decimals:

1. Line up the two decimals according to place value. An easy way to do this is to make sure the decimal points are on top of each other.
2. Compare place values until a difference is found. Start with the whole number parts. If those are the same, check the tenths place of each. If they are the same, check the hundredths, then the thousandths, etc. Keep checking until you find a place value where the digits are not the same.
3. Determine which is larger.

In the place value where you find the difference, the larger digit tells you which number is larger.

Example: Compare 1.1324549 and 1.1324639.

Solution

Step 1: Line up the two numbers by their decimal points.

$$\begin{array}{l} 1.1324549 \\ 1.1324639 \end{array}$$

Step 2: Compare place values until a difference is found. Circle the difference. Notice that it is in the hundred-thousandths place.

Step 3: Determine which is larger. You circled the digits 6 and 5. $6 > 5$, so $1.1324639 > 1.1324549$.

Now you try!

3. Compare the following decimals using < , > , or =.

a. 12 and .13

b. 102 and .13

c. and .999

d. 16.82736 and 16.82747

Terminal and Repeating Decimals

Decimals are created by performing the division in fractions. Divide the denominator of a fraction into its numerator. The result will be a decimal.

Example: Look at the fraction, $\frac{1}{2}$. When division is performed, it becomes the decimal .5.

$$1 \div 2$$

$$\frac{1}{2} = \qquad\qquad = 2\overline{)1}$$

$$\begin{array}{r} . \\ 2\overline{)1.0} \end{array}$$

Make 1 into 1.0, and put a decimal point directly above the division bar too.

$$\begin{array}{r} 0. \\ 2\overline{)1.0} \\ \underline{-0} \\ 1 \end{array} \qquad \begin{array}{r} 0. \\ 2\overline{)1.0} \\ \underline{-0} \\ 10 \end{array} \qquad \begin{array}{r} 0.5 \\ 2\overline{)1.0} \\ \underline{-0} \\ 10 \\ \underline{-10} \\ 0 \end{array}$$

Now just divide, as if it's the whole number 10.

Example: Write $\frac{5}{8}$ as a decimal.

Solution $\frac{5}{8} = 5 \div 8$

$8\overline{)5}$

$$\begin{array}{r} 0. \\ 8\overline{)5.0} \\ -\underline{0} \\ 5 \end{array} \qquad \begin{array}{r} 0. \\ \overline{)5.0} \\ -\underline{0} \\ 50 \end{array} \qquad \begin{array}{r} 0.6 \\ 8\overline{)5.0} \\ -\underline{0} \\ 50 \\ -\underline{48} \\ 2 \end{array} \qquad \begin{array}{r} 0.6 \\ 8\overline{)5.00} \\ -\underline{0} \\ 50 \\ -\underline{48} \\ 20 \end{array}$$

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ -\underline{0} \\ 50 \\ \underline{48} \\ 20 \\ -\underline{16} \\ 40 \\ -\underline{40} \\ 0 \end{array}$$

If there is a remainder, create a decimal and keep adding zeros to the dividend until there is no remainder.

In both of the above examples, the resulting decimals had an end. Decimals that have an end are called terminating decimals. Not all decimals are terminating. Not all of them have an end. Take a look at the example on the next page.

Example: A sign in a store says "*Markers: 3 for $1.00 or 1 for $ .35*". Which one is the better deal?

Solution: You must figure out what the cost of one marker is in each deal and compare the prices. In the first deal, 3 markers are offered for one dollar. The price of one marker can be shown as $\frac{1}{3}$ of a dollar. How much is this? Remember: Fractions mean division.

$$\frac{1}{3} = 1 \div 3 \quad = 3\overline{)1.0000}^{\,0.3333}$$

$$\begin{array}{l} \underline{.9} \\ .10 \\ \underline{.09} \\ .010 \\ \underline{.009} \\ .0001 \end{array}$$

No matter how long you keep dividing, this decimal will never end. You will keep adding on another 3 forever! A decimal that does not end is called a repeating decimal.

To show that a decimal never ends put a bar over the part that repeats. In the example above, 0.33333333333333… is written as $.\overline{3}$. Because you are dealing with money, you need to round this decimal to the hundredths place. (Rounding will be explained later in this lesson.)

.3333333. . . or $.\overline{3}$ rounds to .33.

Thus, one marker from the first deal will cost $.33. That is a cheaper price than the $.35 of the other deal.

Another example of a decimal that never ends is 0.6437121212121212121212…

It is written as $0.643\overline{12}$. Notice that the bar only goes over the numbers that repeat.

That makes the decimal easier to read.

- ✓ A decimal that ends is called a **terminating decimal**. For instance, .173 and 33.2 are terminating decimals.
- ✓ A **repeating decimal** is a decimal that has an <u>infinite number of digits</u>. The digits continue in a set <u>pattern</u>.
- ✓ For example, $.3333333... = .\overline{3}$, and $.473473473473473... = .\overline{473}$ are repeating decimals.

Any fraction can be made into either a terminating or a repeating decimal!

Example: Write $\frac{4}{5}$ as a decimal.

Solution: Use long division.

$$\begin{array}{r} 0.8 \\ 5\overline{)\ 4.0} \\ -\underline{4.0} \\ 0 \end{array}$$

$\frac{4}{5} = 0.8$, a terminating decimal.

Example: A sign in the store reads "***Paper towels, 11 rolls for $3.00***". How much will 1 roll of paper towels cost?

Solution: You must divide 3.00 into 11 equal groups.

```
                 0.27272...
**3 ÷ 11 =**  11)3.00000...
               -2.2
                  80
                -77
                   30
                 -22
                    80
                   -77
                     ⋱
```

As soon as you see the same remainder twice, you know that the decimal is a repeating decimal. You can stop dividing at that point.

The answer: $3 \div 11 = .\overline{27}$

Rounding

In the previous example, you found that one roll of paper towels cost $\$0.\overline{27}$. A repeating decimal is acceptable in mathematics. It does not work for money. Stores cannot charge $0.272727272727. . . for an item. If $0.27 equals 27 pennies, how many pennies equal $0.27272727? Pennies are the smallest division of a dollar. Parts of a penny do not exist.

When dealing with money, repeating decimals are rounded to the nearest hundredth or cent. One one-hundredth of a dollar = one cent = one penny. In the case of the paper towels, $0.27272727 is rounded to $0.27. Thus, the cost of one roll of paper towels is $0.27.

Rule to round a number to a given place value	
1. Look at the number to the right of the place value you are asked to round to.	Round 1.895 to the nearest hundredth.
2. Compare that number to 5.	
a. If that number is less than 5, round down and leave the given place value the same.	1.895
b. If that number is greater than or equal to 5, round up and increase the given place value by 1.	5 = 5 Round up
c. If the number in the given place value is a 9, make it a 0 and increase the value of the number to the left of our given place value by 1.	1.895 rounds to 1.90
d. In equations, rounded answers require a special sign. Use **≈**, not = in the equation.	

Example: Round 173.9378429329 to the nearest <u>tenth</u>.

Solution

Step 1: Look at the number to the right of the tenths place.

173.9<u>**3**</u>76429329

Step 2: Compare the number to 5. Notice that $3 < 5$, so you must round <u>down</u>. Leave the number in the tenths place the same. Your answer is 173.9.

Now you try!

Convert the following fractions to decimals. The decimals may be either terminating or repeating.

4. $\frac{9}{11}$

5. $\frac{11}{8}$

6. $\frac{5}{6}$

Convert the following fractions to decimals. The decimals may either terminate or repeat. Then, round each decimal to the nearest <u>hundredth</u>.

7. $\frac{3}{8}$

8. $\frac{2}{3}$

9. $\frac{5}{11}$

10. Write the decimal with words, then as a mixed number, then as an improper fraction in simplest form.

4756.5

11. Use an inequality sign (> or <) to compare each pair of decimals.

a. 3.425 ____ 6.425 **b.** 1.089 ____ 1.1

c. 0.001 ____ 0.01 **d.** 142.284756 ____ 142.284755

12. Round each decimal to the nearest <u>hundredth</u>.

a. 7.43232 **b.** 14.267239 **c.** 9.473

d. 1.1111111111 **e.** 0.9877654 **f.** $13.\overline{8}$

13. Write the following decimals so that their place values are lined up.

24971894781.34 and 32.823743239

14. Write the amount as a decimal part of a dollar. (Hint: think of how many <u>cents</u> each equals.)

a. 1 quarter $ _____ **b.** 4 nickels $ _____ **c.** 89 pennies $ _____ **d.** 14 dimes $ _____

End of Lesson 6

Lesson 7

Percents

Word to know:

√ **percent**

A ***percent*** is the comparison of any number to 100. Let's take a closer look at percents.

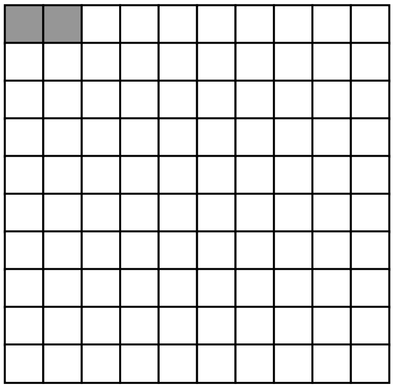
You are at the checkout at the store. You are buying a pack of 3 markers for $1 and a pack of gum for $0.32. Your total is $1.32. You notice a sign that says, "*8% sales tax*". What does this mean? How much will you pay?

Start with 8%. 8% is read as "eight percent."

✓ A percent is a comparison of any number to 100. The symbol % means $\frac{1}{100}$. Percents can be changed into both fractions and decimals.

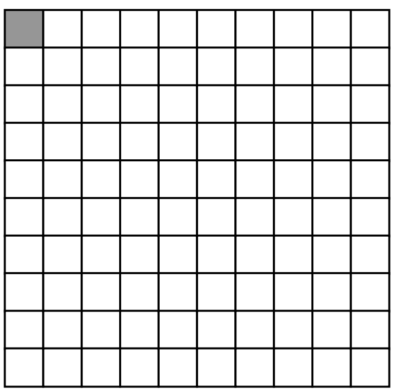
For example, 3% means , $\frac{3}{100}$ or .03.

Think about what that means. Percents compare everything to 100. The model below equals 100. It has been divided into 100 equal-sized boxes. How would you show 2?



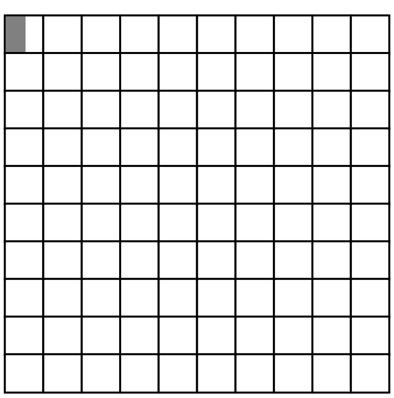
If 100 boxes equal 100, each box is worth 1. To show 2, shade two boxes. The shaded boxes are 2 out of 100. 2 compared to 100 equals 2%. Or, 2 is 2% of 100. The 2 shaded boxes represent 2% of the model.

What if the same model of 100 boxes is equal to <u>200</u>? How would you show 2?



Now, the model equals 200. Each of its 100 little boxes is worth 2. (100 x 2 = 200.) In order to show 2, you only need to shade 1 box. 1 box out of 100 means 1 compared to 100, or 1%. The value of each box is 2. Thus, 2 is 1% of 200.

The model below still equals 200. This time, it shows the value of 1.

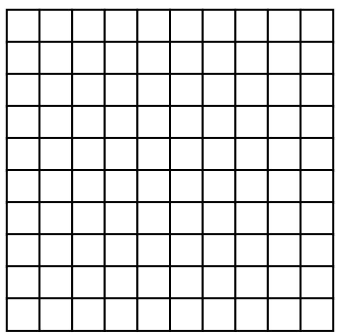


Each little box is worth 2. One is half of 2. It is shown by shading half of a little box. One half compared to 100 equals 0.5%. Thus,1 is 0.5% of 200.

Now you try! For each question below:

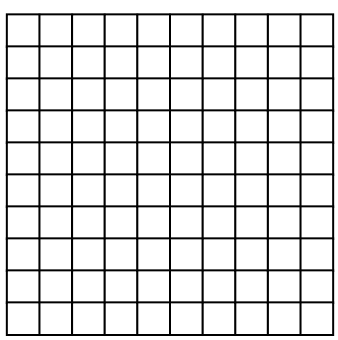
- ✓ Note the value of the whole model.
- ✓ State how much each little box is worth.
- ✓ Shade in the required value based on the model.
- ✓ State the percent it takes up.

1. The whole is 100, so each little square is worth _______________. Now shade in 37.



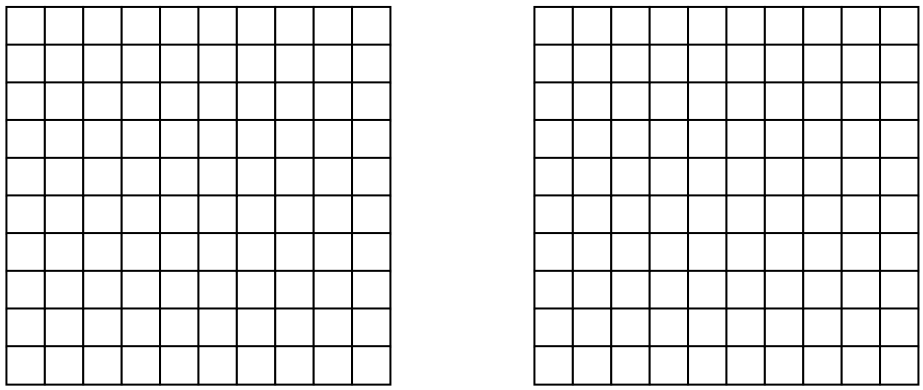
Percent that 37 is of 100: ______________________

2. The whole is 200, so each little square is worth ________________. Now shade in 75.



Percent that 75 is of 200: _____________________

3. Each whole represents 50. The value of one little box is ________________.
Now shade 60. (*Hint:* There are two wholes because 60 is bigger than 50).

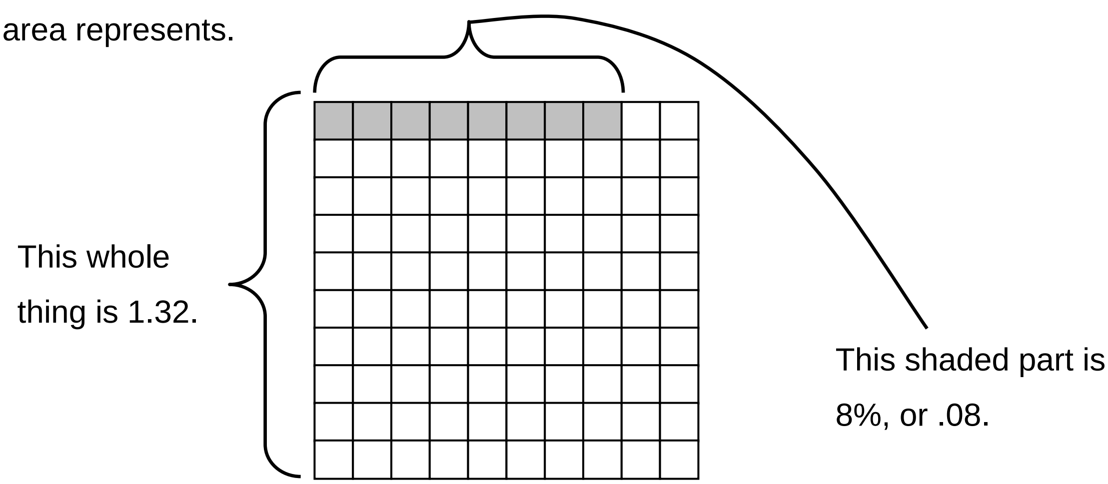


The percent 60 is of 50:__________________

Now you can figure out the sales tax problem from the beginning of this lesson. There is an 8% sales tax on your \$1.32 purchase of markers and gum.

8% really means $\frac{8}{100}$ or .08 . So the tax is .08 <u>of</u> 1.32.

The model below represents the whole, 1.32. You want to know how much the shaded area represents.



The 8 shaded boxes represent the tax on the items, or .08 of 1.32 .

FACT

The word "of" implies multiplication. For example, half of four means $\frac{1}{2} \times 4$. Thus, .08 of 1.32 = .08 x 1.32.

Follow these steps to multiply decimals.

(1) Count the number of decimal places in each factor and add them together.

$$\underset{2}{.08} \quad \times \quad \underset{2}{1.32}$$

2 + 2 = 4 decimal places

(2) Multiply as if you are multiplying whole numbers.

$$\begin{array}{r} 132 \\ \times \quad 8 \\ \hline 1056 \end{array}$$

(3) Place the decimal point in your answer by counting from the farthest right digit. Use the answer you found in Step 1.

1 0 5 6 = **.**1056

4 3 2 1
(decimal places)

8% of $1.32, then, is $0.1056. Because you are dealing with money, you must round to the nearest cent or hundredth. Rounded to the nearest cent, the sales tax equals $0.11 .

Find the total cost of your purchase. Add the cost of the items plus the sales tax.

1.32 + .11 = 1.43

Total cost of purchase = $1.43

When you calculate the amount of sales tax of a purchase, you are taking the percentage of a number. Taking the percentage of a number means multiplying the number by a percent.

Rule to find a percentage of a number:	Find 13% of 75.
1. Change the percent to a decimal. Move the decimal point two places to the left.	13% = 0.13
2. Multiply the decimal by the number.	75×0.13 $= 9.75$

Example: What is 15% of 13?

Solution

Step 1: $15\% = 15 \div 100 = 0.15$

Step 2: 0.15 x 13

$$\begin{array}{r} 15 \\ \times\ 13 \\ \hline 45 \\ +\ 150 \\ \hline 195 \end{array}$$

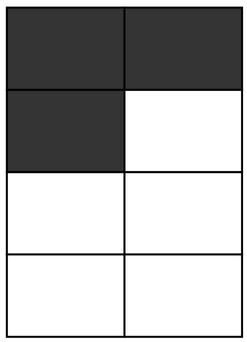
1.95

Sometimes, you are given two numbers and asked to find a percent.

Example: What percent of 8 is 3?

Solution: Flip the words and turn the question into a statement. You want to know what percent 3 is out of 8.

This is 3 of 8.



$$\frac{3}{8}$$

As a decimal, this is

$$\begin{array}{r}
.375 \\
8\overline{)\,3.000} \\
\underline{-2.4} \\
60 \\
\underline{-\;56} \\
40 \\
\underline{-\;40} \\
0
\end{array}$$

Now, you need to convert the decimal to a percent.
Remember: Percents compare numbers to 100.

Multiply the decimal .375 by 100 to get its percent form.

$$\begin{aligned}
&.375 \\
&= .375 \times 100\% \\
&= 37.5\%
\end{aligned}$$

You just found that 3 <u>is</u> 37.5% <u>of</u> 8.

Rule to find what percent a number is of another number:

1. Convert the statement into a fraction of the form "is over of," or $\frac{\text{“is”}}{\text{“of”}}$.
2. Convert the fraction to a decimal using long division.
3. Convert the decimal to a percentage. Multiply the decimal by 100, and write a % sign at the end of the number.

Example: To the nearest percent, what percent of 11 is 3?

Solution

Step 1: $\frac{\text{"}is\text{"}}{\text{"}of\text{"}}$

Here, 3 "is" some percentage "of" 11. So the fraction is $\frac{\text{"}is\text{"}}{\text{"}of\text{"}} = \frac{3}{11}$

Step 2: Convert to a decimal.

$$
= 11\overline{)3.000\ldots} \quad (\text{quotient } .272\ldots)
$$

$$
\begin{array}{r}
-\ 2.2 \\
\hline
80 \\
-\ 77 \\
\hline
30 \\
-\ 22 \\
\hline
\ddots
\end{array}
$$

$$
= .\overline{27}
$$

Step 3: Convert to a percent

$$.\overline{27} \times 100\%$$

$$= 27.\overline{27}\%$$

Step 4: Round to the nearest percent

$$27.\overline{27}\% \approx 27\%$$

Example: Convert 38% to a decimal.

***Solution*:** Going back to the meaning of percent,

$$38\% = 38 \times \frac{1}{100}$$

$$= \frac{38}{100}$$

$$= .38$$

You simply moved the decimal point two spaces to the left!

Rule to convert from a decimal to a percent: Move the decimal point two places to the right Put a % sign at the end.	0.43 4 3. 43%
To convert from a percent to a decimal: Drop the % sign. Move the decimal point two places to the left.	17% 17 0.1 7

Now you try!

4. What percent of 5 is 4?

5. What is 14% of 2,350?

6. Convert the percents to decimals.

a. 10% **b.** 25% **c.** 19%

d. 61% **e.** 72.1% **f.** 129%

7. Convert each decimal into a percent.

a. 0.14 **b.** 0.10 **c.** 0.78

d. 0.01 **e.** 1.02 **f.** 0.75

g. 0.003 **h.** 2.45

8. Find:

a. 10% of 30 **b.** 15% of 75

c. 50% of 47 **d.** 25% of 20

9. Convert the following fractions to percents. Round to the nearest hundredth, if necessary.

a. $\frac{3}{11}$ **b.** $\frac{1}{4}$

c. $1\frac{1}{5}$ **d.** $\frac{7}{8}$

e. $\frac{2}{12}$ **f.** $\frac{9}{10}$

10. Solve the following percent word problems.

a. 5 is what percent of 80? **b.** 20 is what percent of 22?

Notes:

End of Unit 2