



Two Plus You

Unit

2



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## Factors and Multiples

**Words to know:**

- ✓ factors
- ✓ multiples

**F***actors* are the numbers you multiply together to get another number. Let's take a closer look at factors.

**Example:** One day, you and your friends decide to play basketball. There are 12 people all together. How many different teams can be made using 12 people?

You could think of this problem using multiplication.

You and your friends are 1 group of 12 people:  $1 \times 12 = 12$ .

You could separate into 2 teams of 6 people:  $2 \times 6 = 12$ .

Or 3 teams of 4 people:  $3 \times 4 = 12$ .

You could also make 4 teams of 3, 6 teams of 2, or even 12 teams of 1.  $4 \times 3 = 6 \times 2 = 12 \times 1 = 12$

The whole numbers that were used to multiply to 12 are:

1, 2, 3, 4, 6, 12

They are all factors of 12. Notice that 12 is divisible by all of these numbers.

Read the following definition carefully.

- ✓ When whole numbers, other than zero, are multiplied together, each number is a factor of the product.

**Example:** 2 and 7 are factors of 14, because  $2 \times 7 = 14$ . Similarly, if a whole number divides evenly into a number, the divisor and quotient are factors of that number. 2 and 7 are factors of 14, because  $14 \div 7 = 2$ .

In the basketball problem, two of the different ways of grouping were  $3 \times 4$  and  $4 \times 3$ .

When you list the factors of a number, count each factor only once. Do not write the same factor twice. Thus, 3 and 4 are listed only once as factors of 12.

## Now you try!

1. List all the factors of the following numbers:

a. 24

b. 10

c. 36

Two other factors of 12 are 2 and 6. Notice that 2 has no factors other than 1 and itself, 2. Because of this fact, 2 is defined as a prime number.

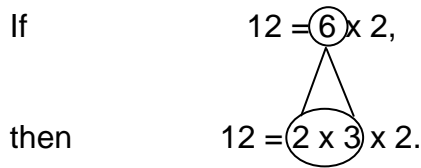
✓ A number is **prime** if its only factors are 1 and itself. For example, 5 is prime because the only numbers that divide into it evenly are 1 and itself.

The number 6 is not prime. It has more factors than 1 and itself. All the numbers that divide evenly into 6 are: 1, 2, 3, 6

Six has more factors than just 1 and itself. It is called a composite number. It is a composition of many factors.

✓ A **composite number** is a whole number greater than 1 that has factors in addition to 1 and itself. For example, 4 is composite because it has the factors 1, 2, and 4.

The composite number 6 can be written as the product of two of its factors,  $6 = 2 \times 3$ .



**FACT**

*The number 1 is neither prime nor composite!*

$2 \times 3$  is equal to 6. It can be substituted for 6 in the equation.

You can see that the number 12, written as

$$12 = 2 \times 3 \times 2,$$

has two factors of 2 and one factor of 3.

Written this way, all the factors of 12 are prime numbers.

**FACT**

*Every whole number can be written as the product of prime factors! This is a very special property called the Fundamental Theorem of Arithmetic!*

One way to factor a number into primes is by using a factor tree.

**Example:** Write 72 as a product of its prime factors.

**Solution:** You can solve this with the factor tree method.

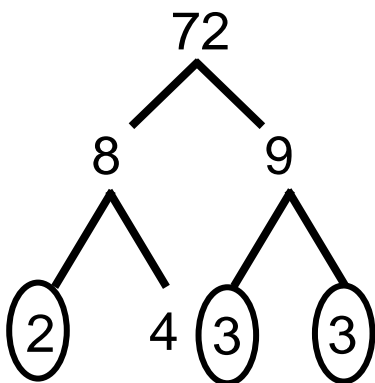
**Step 1:** Write the number you want to factor.

72

**Step 2:** Draw two “branches” down from that number. Put two of its factors at the end of the branches. Never use the factor 1.

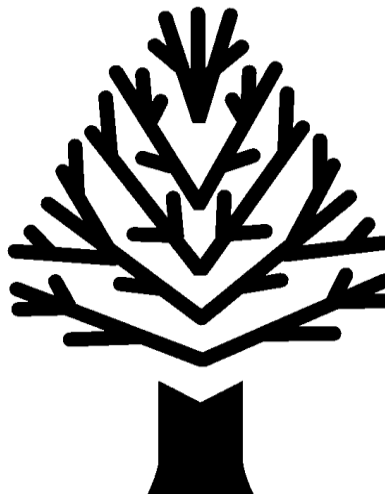
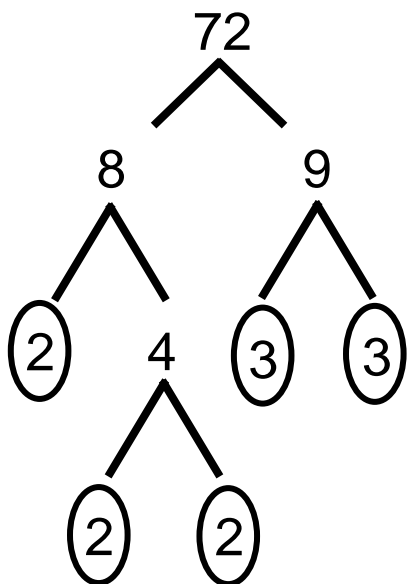


**Step 3:** Continue to draw branches off each factor, until you have reached a prime number. Circle the prime factors as they occur.



Factor 8 and 9, and circle the prime factors.

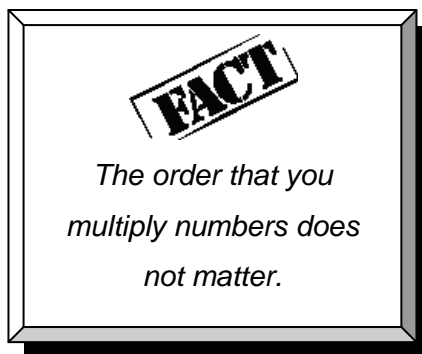
Now, factor the 4 and circle its prime factors.



Your factor tree is now complete, but you are not finished yet!

**Step 4:** Write the answer as a product of prime numbers. The final product is equal to:

$$72 = 2 \times 3 \times 3 \times 2 \times 2 = 2 \times 2 \times 2 \times 3 \times 3$$





**Rule to factor a number:**

1. Write the number you wish to factor at the top.
2. Draw two branches below the number. Write the factors of the number at the end of the branches. Do not use 1 or the number as factors unless there are no others.
3. Circle any prime numbers. Continue to factor the composite numbers until all the factors are prime. Circle them.
4. Write the number as a product of its prime factors.

**Now you try!**

2. Factor each number using a factor tree. Then write the number as a product of prime factors.
  - a. 64
  - b. 100
  - c. 36

The factor-tree method is very useful for finding the prime factors of a number. You can also use it to find factors that are common to two (or more) numbers.

Comparing the factors of two (or more) numbers:

- ✓ Factors that are not shared are called *unique factors*.
- ✓ Factors that the numbers share are called common factors.
- ✓ The largest factor two (or more) numbers share is their greatest common factor, or their **GCF**. For instance, 2 is the GCF of 4 and 6.

**Example:** Find the greatest common factor of 90 and 135.

**Solution:** The steps to solving this problem are:

- list the factors of each number,
- find their common factors, and
- determine which factor is the largest.

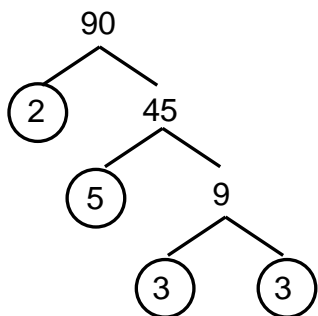
Factors of 90:	1,	2,	3,	5,	9,	10,	18,	30,	45,	90
Factors of 135:	1,	3,	5,	9,	15,	27,	45,	135		

**You can see that 45 is the GCF of 90 and 135.**

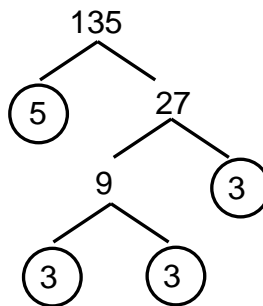
The above method has its problems. It was not efficient to list every factor of 90 and 135. It is also easy to miss factors, and make mistakes. There is an easier way to solve this problem. It uses factor trees and Venn diagrams.

The other method:

**Step 1:** Factor each number using a factor tree. Rewrite it as a product of prime factors.

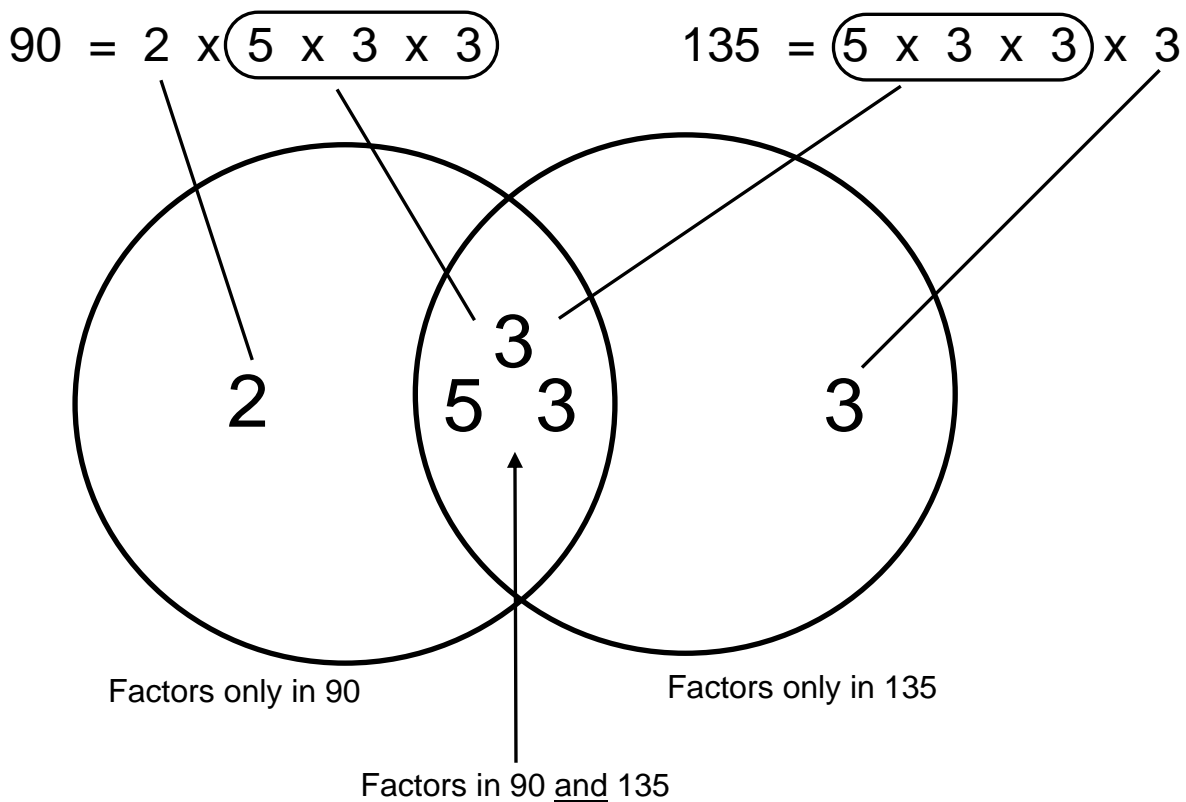


$$90 = 2 \times 5 \times 3 \times 3$$



$$135 = 5 \times 3 \times 3 \times 3$$

**Step 2:** Sort using a Venn diagram.



**Step 3:** The first method showed that the GCF of 90 and 135 is 45.

Look at the common prime factors of 90 and 135.

They are 5, 3, and 3. Notice that  $5 \times 3 \times 3 = 45$ .

The Venn Diagram method gave you the same answer as the first method. And, it is a good way to avoid forgetting factors!

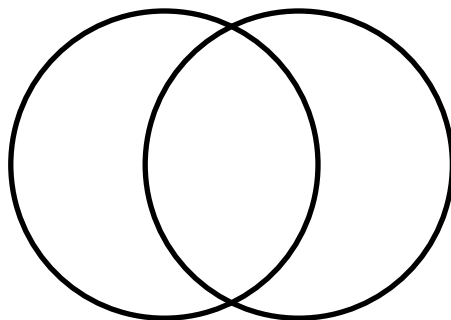
**Rule to find the greatest common factor (GCF):**

1. Factor each number and rewrite it as a product of prime factors.
2. Organize the factors of each number using a Venn diagram.
3. Multiply all of the numbers in the center section of the Venn diagram together.  
This is the GCF.

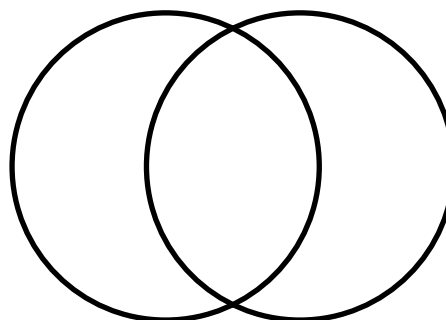
**Now you try!**

3. Find the greatest common factor for each pair of numbers.

a. 72 and 108



b. 70 and 315



## Multiples

You have invited your friends to a barbecue at your house. You need to get hot dogs and rolls for the barbecue. Hot dogs come in packs of 6. Hot dog rolls come in packs of 8. You want the number of hot dogs and rolls to be the same. You will need to buy multiple packs of hot dogs and rolls until you have the same number of each. How can you find out the number of packs of hot dogs and hot dog rolls you need to buy?

This is a multiplication problem. The number of hot dogs you buy will equal the number of packs times 6 hot dogs each. The number of hot dogs you might buy is

$$6 \text{ hot dogs} \times 1 \text{ pack} = 6$$

$$6 \text{ hot dogs} \times 2 \text{ packs} = 12$$

$$6 \text{ hot dogs} \times 3 \text{ packs} = 18$$

$$6 \text{ hot dogs} \times 4 \text{ packs} = 24$$

$$6 \text{ hot dogs} \times 5 \text{ packs} = 30$$

$$6 \text{ hot dogs} \times 6 \text{ packs} = 36$$

The number of rolls you might buy is

$$8 \text{ rolls} \times 1 \text{ pack} = 8$$

$$8 \text{ rolls} \times 2 \text{ packs} = 16$$

$$8 \text{ rolls} \times 3 \text{ packs} = 24$$

$$8 \text{ rolls} \times 4 \text{ packs} = 32$$

$$8 \text{ rolls} \times 5 \text{ packs} = 40$$

$$8 \text{ rolls} \times 6 \text{ packs} = 48$$

Hot dogs come in multiples of 6, and buns come in multiples of 8.

- ✓ A multiple of a number is the product of that number and any whole number besides zero. For example, 20 is a multiple of 4. ( $4 \times 5 = 20$ )

As you can see, 6 and 8 have some multiples in common.

Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48.  
 Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64...

You can see that both 6 and 8 have the multiples 24 and 48. These are common multiples. In terms of your barbecue, common multiples mean you will have the same number of hot dogs as rolls. The smallest multiple these numbers share is the least common multiple.

- ✓ The smallest multiple two numbers share is called the least common multiple, or LCM.

The least number of hot dogs and rolls you should get is 24. If you buy 4 packs of hot dogs and 3 packs of rolls, you will have 24 of each.

$$4 \times 6 = 3 \times 8 = 24$$

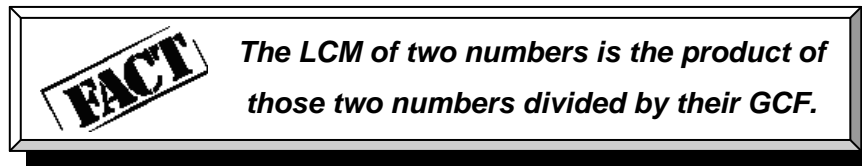
**Example:** Find the LCM of 12 and 20.

**Solution:** List the multiples of each number.

Multiples of 12 are: 12, 24, 48, 60, 72, ...

Multiples of 20 are: 20, 40, 60, 80, 100, ...

**The least common multiple is 60.**



Check to see if this fact works.

Factors of 12: 1, 2, 3, 4, 6, 12

Factors of 20: 1, 2, 4, 5, 10, 20

**4 is the GCF.       $12 \times 20 = 240$**

**$240 \div 4 = 60$**

It works! You can use this fact to check your answer when finding the LCM of two numbers.

### Now you try!

4. Find the least common multiple for each pair of numbers.

a. 8 and 16

b. 24 and 84

c. 13 and 17

Notes:

∞ End of Lesson 4 ∞