Lesson 2

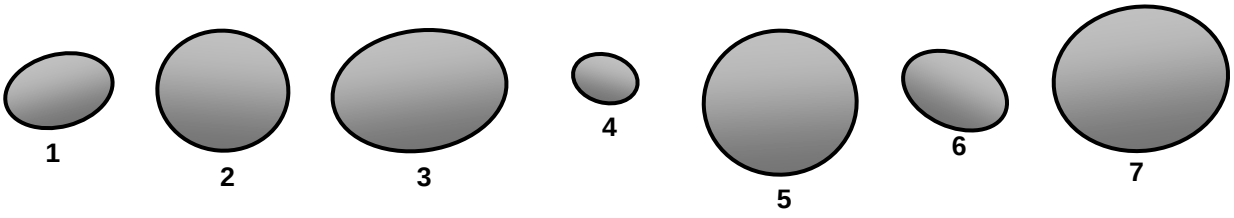
Addition and Subtraction With Integers

Words to know:

- ✓ **addition**
- ✓ **subtraction**

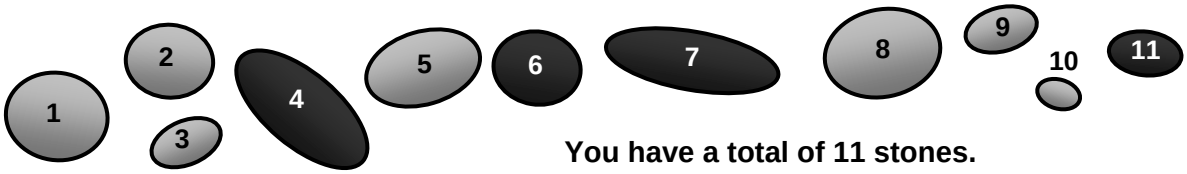
Addition is the process used to count the number of objects in two or more groups. Imagine you are walking on a beach, picking up stones. You pick up some dark stones and count them.

There are four dark stones. You then see how many light colored stones you can find and count those.



You have four dark stones and seven light stones.

How many stones do you have all together? Count them.



You have a total of 11 stones.

You used addition to find the sum of the number of dark stones and light stones. To add 4 and 7, you started at 4, counted 7 more, and ended with 11. You could have also started with 7 and counted 4 more; the answer would have been the same.

- ✓ Addition is used to count the number of objects in two or more groups.
- ✓ The symbol used for addition is "+".
- ✓ The sum is the number you get when you add two or more numbers together.

For example: 2 + 3 = 5. 9 plus 7 is 16. The sum of 6 and 0 is 6.

There are several ways to show addition in math.

Example: Adding four dark and seven light stones could be written like this:

$$4 + 7 = 11, \quad \text{or} \quad \begin{array}{r} 4 \\ +\,7 \\ \hline 11 \end{array}$$

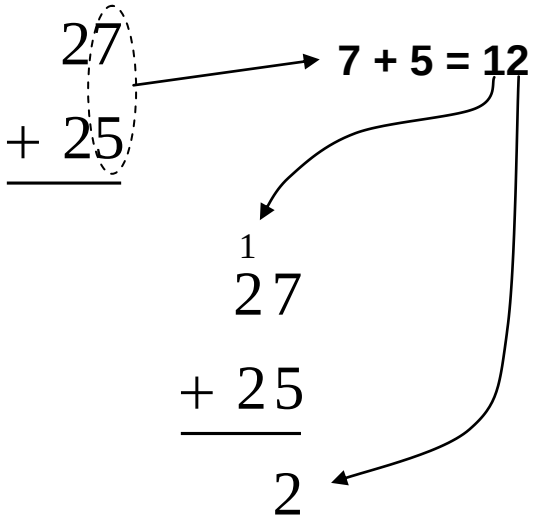
Both ways of writing the addition problem are correct. The second way is more helpful when adding larger numbers. Here is another example.

Example: Jahmel threw a ball 27 feet. He picked it up and threw it again. The second time it went 25 feet. How many total feet did Jahmel throw the ball? Or, what is the sum of the distances that Jahmel threw the ball?

Solution: Use addition to solve this problem.

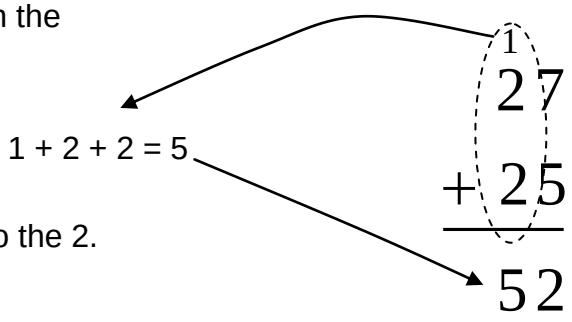
Step 1: Line the numbers up on top of each other as shown below.

Step 2: Add the digits farthest to the right first.



Step 3: Put the 2 below, and carry the 1 to the next place, as follows.

Step 4: Now add each digit in the next column.



Step 5: Write this sum next to the 2.

So, Jahmel threw the ball a total of 52 feet.

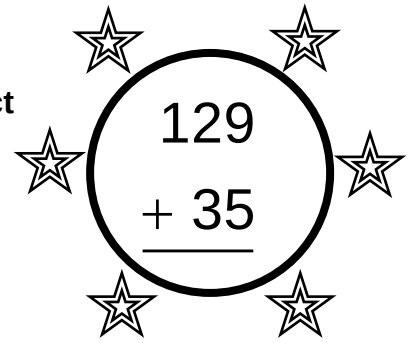
Rule to add two or more numbers:

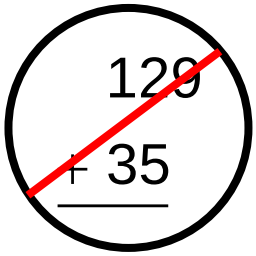
1. Write the numbers. Put the digits at the far right of each number on top of one another.
2. Add all the digits. Start with those at the far right.
   a. If the first sum is ten or more, write the digit in the ones place directly under the digits you added.
   b. Write the number in the tens place above the next column to the left.
3. Add the digits in the column to the left of the ones you just added.
   a. If the sum is ten or more, repeat step 2, a. and b.
4. Repeat this process, right to left, until every column of digits has been added.

Example: Tony filled up his gas tank for $35. Later that day, his car broke down. It cost $129 to repair it. How much money in total did Tony spend on his car that day?

Solution: To solve this problem, you must add 35 and 129. When you line up the numbers, make sure the digits farthest to the right are lined up.

Correct





Incorrect

Now, follow the steps from before. Work from right to left.

The final sum looks like this.

$$\begin{array}{r} {\scriptstyle 1} \\ 129 \\ +\ 35 \\ \hline 164 \end{array}$$

Tony spent a total of $164 on his car that day.

This method also works for adding more than two numbers.

Now you try!

1. Find the sums

a. 1 + 2 =

b. 7 + 2 =

c. 11 + 4 =

d. 4 + 6 =

e. 7 + 6 =

f. 7 + 8 =

g. 14 + 7 =

h. 30 + 40 =

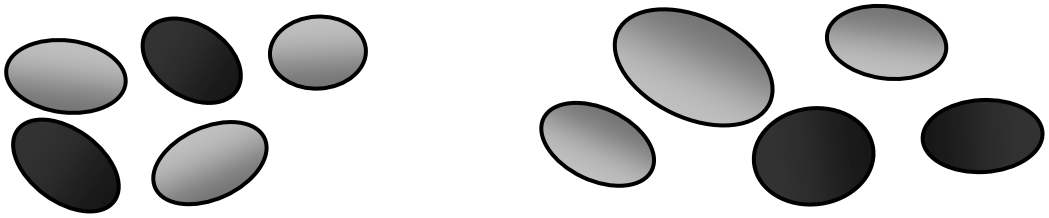
i. 179 + 5 =

2. Pedro picked 1,247 peaches on Monday and 989 on Tuesday. How many peaches did he pick all together?

3. Find the sum. 124 + 65 + 4

Subtraction

You are back on the beach. This time, you have picked up ten stones. You decide to throw five of them into the water, one at a time. After you throw the first stone, you count down one from ten. That leaves nine stones. Then you throw another stone. Now, you have eight stones left. Then seven, and finally six stones are left in your hand.



stones thrown into the water

The process of taking away is called ***subtraction***.

- ✓ Subtraction is used when things are taken away from a group.
- ✓ The symbol for subtraction is a dash, "–". It is called a minus sign.
- ✓ The answer to a subtraction problem is called the difference.
- ✓ Three ways to show subtraction:
- ✓ 5 – 3 = 2. 14 minus 2 is 12. The difference between 21 and 7 is 14.

Example: What is the difference between 8 and 5?

Solution: When a question asks you to find the <u>difference</u>, that means subtraction. To find 8 – 5, start from 8 and count backwards 5.

8	7	6	5	4	3
	1	2	3	4	5

So, 8 – 5 = 3

There is another, simpler method to find the difference between two numbers. It uses addition.

Example: Adrian bought groceries that cost $23. He gave the cashier $30. How much change should Adrian receive?

Solution: To find how much change Adrian will receive, you must find the difference between 30 and 23. One way to find this is to ask the question, "23 plus what will give me 30?" In math terms:

$$23 + \_\_\_ = 30$$

Count up from 23 to 30, and keep track of how many that is.

24	25	26	27	28	29	30
(1	**2**	**3**	**4**	**5**	**6**	**7)**

The answer is seven. $23 + \underline{7} = 30$.

Adrian will receive $7 in change.

This method is very helpful for doing subtraction in your head. Many people use their fingers to subtract with this method.

What if you have to subtract large numbers? Will counting and using your fingers work?

Example: Pedro has a bag with 150 candies in it. He gives away 72 pieces of candy on Halloween. How many pieces of candy does he have left?

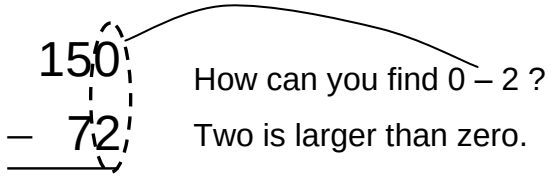
Solution: Phrases like *give away*, *less, take away*, or *minus* mean subtraction. In this case, the problem says that Pedro gives away 72 pieces from his 150 pieces of candy. That means subtraction: 150 – 72

These two numbers are large. To subtract them, use the same method you used with adding large numbers. Follow the steps below.

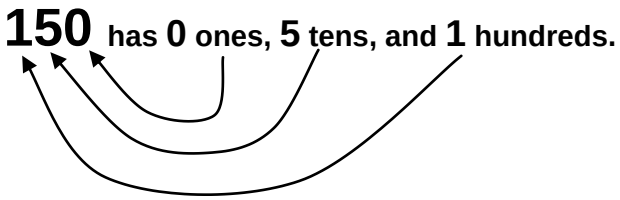
$$\begin{array}{r} 150 \\ -\ 72 \\ \hline \end{array}$$

Step 1: Write the numbers on top of one another, with the digits on the right perfectly lined up.

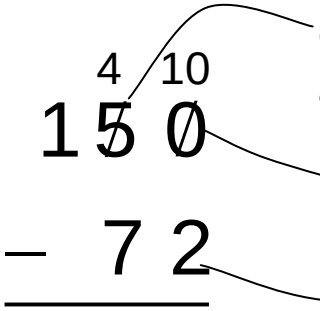
Step 2: Starting with the digits on the right, subtract the bottom one from the top one.



You must use the *borrowing method*. Look at the number 150 and think back to place values.



You can borrow one of the tens from the five tens to perform the subtraction. One ten is worth ten ones. (Think about money. A ten-dollar bill can be exchanged for ten one-dollar bills.) Take 1 ten from the 5 and exchange it for ten ones.

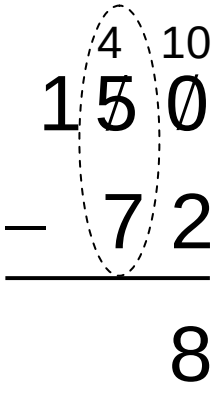


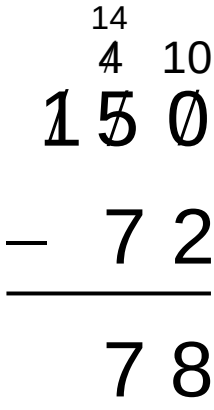
Cross out the 5 and subtract 1 from it. 5 – 1 = 4. Write the 4 above the crossed out 5.

Now cross out the 0, and add 10 to it. 0 + 10 = 10. Now you can subtract the 2 from ten.

Next, look at the 4 and the 7 in the middle column.

Since 7 is larger than 4, you must borrow again,





The finished problem will look like this.

Pedro has 78 pieces of candy left.

Rule to subtract two numbers:

1. Write the numbers with the digits at the far right directly on top of one another.
2. Starting with the digits farthest to the right, subtract the bottom from the top.
   a. If the bottom digit is larger than the top digit, borrow one unit from the tens place of the top number. Subtract one from it, and add ten to the ones place of the top number. Then subtract the ones.
3. Subtract the digits in the tens places.
   a. Borrow from the hundreds place, if the bottom digit is larger than the top in the tens place.
4. Repeat this process until every column of digits has been subtracted.

Now you try!

4. Find the differences

a. 3 – 2 =	**b.** 7 – 4 =	**c.** 9 – 3 =
d. 6 – 5 =	**e.** 8 – 1 =	**f.** 11 – 3 =
g. 15 – 7 =	**h.** 13 – 11 =	**i.** 6 – 6 =

5. Solve the next subtraction problems by rewriting them as addition problems (*Ex:* 7 **–** 4 = __ should be rewritten as 4 + __ = 7)

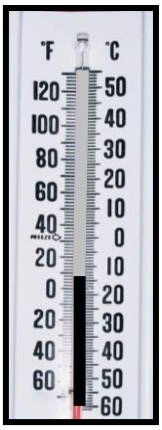
a. 17 – 13 =	**b.** 12 – 9 =	**c.** 56 – 52 =
d. 27 – 22 =	**e.** 13 – 12 =	**f.** 54 – 24 =

6. Tami is saving money for a trip to Hawaii over spring break. The trip will cost $1745. She has saved $1290 for the trip so far. How much more money must she save? _____

Using Number Lines and Integer Chips

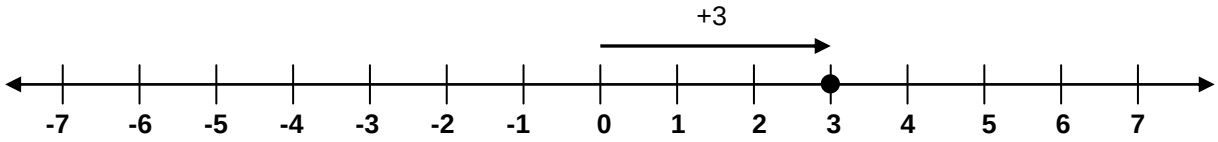
Number lines and integer chips can be used to add and subtract. You may find them useful in real-life situations.

Example: One morning you check the temperature outside. The thermometer has a reading of three degrees. The temperature is supposed to be five degrees colder the next day. What temperature will it be the next day?



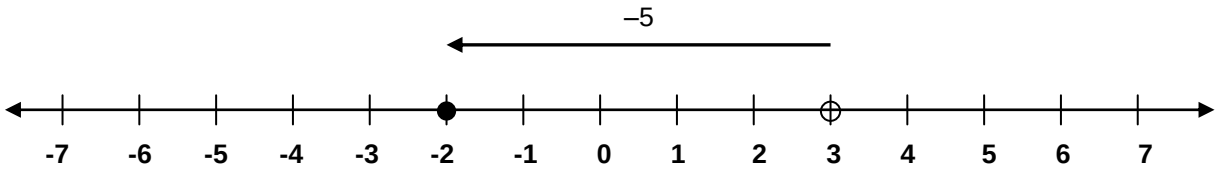
We are now talking about <u>subtracting</u> integers. To find the predicted temperature for the next day, you need to find 3 degrees <u>minus</u> 5 degrees, 3 – 5. To think about this problem, use something you are familiar with. What about a number line?

In this problem, the temperature starts at positive 3 degrees. Plot that on the number line.



When there is no sign in front of a number, it is positive.
3 is a positive number.

The temperature is supposed to go down or decrease by five degrees. In math, "decrease" means subtraction. On a number line, go left for subtraction. Go right for addition. Count five numbers to the left of 3. Put a dot at the new number.



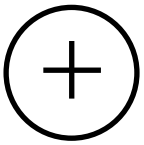
You end up at –2, so 3 – 5 = –2.
The new temperature will be –2 degrees.

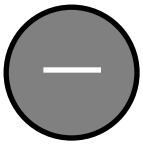
Solving a problem in this way is called the number-line method.
Review the steps you followed.

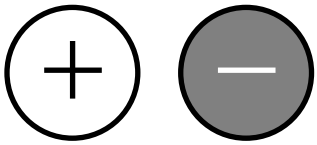
- ✓ Starting at zero, you found the number that began the problem (3).
- ✓ You put a point at its place on the number line.
- ✓ Then, you drew an arrow just above the line, from zero to that number.
- ✓ To subtract, you counted to the left 5 numbers and put a dot at the new number (–2).
- ✓ Then, you drew an arrow, just above the line, from the first number to the new one.

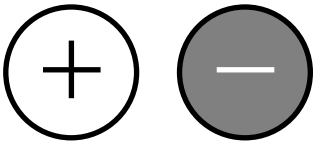
The solution to the problem is the number you ended on: –2.

Another way to solve this problem is the integer chip method. There are two different types of integer chips:

The positive chip  = +1

And the negative chip  = –1

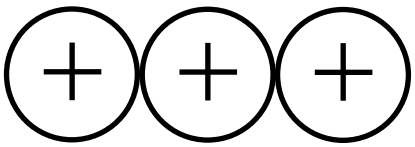
When they are combined,  they equal 0

Since  = –1 + 1 = 0

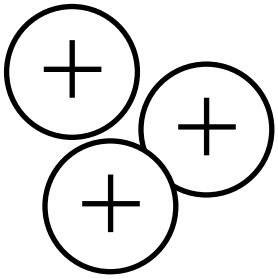
They cancel each other out. To show this, cross them out whenever you see them together.

Now try the temperature problem again using this method.

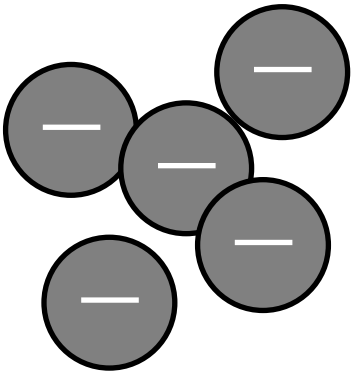
The problem started at positive three degrees. Show this with 3 positive chips.



Then, the temperature dropped 5 degrees. Add 5 negative chips to the pile.



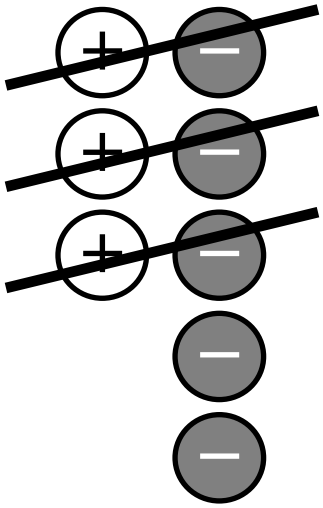
+3



–5

Remember: One positive chip and one negative chip will cancel each other out. ⊕⊖ = 0. Regroup the chips. Make every positive and negative match you can.

Cancellation!

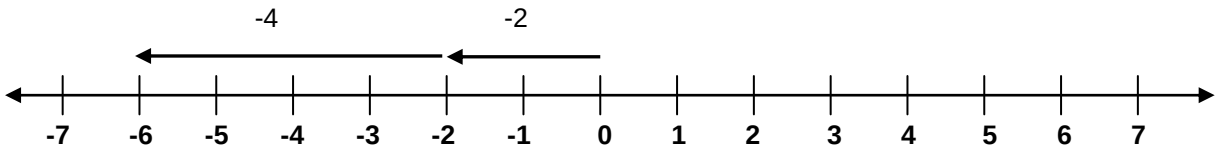


You can see that 2 negative chips are left.

The temperature tomorrow will be –2 degrees.

Here is another example: What is –2 – 4?

Solution: Use a number line. You have a negative number minus another number. "Minus" means subtraction. You will be moving to the left. Find –2 first. Start at zero on the number line and move two spaces to the left. Now, subtract 4. Move another four spaces to the left.

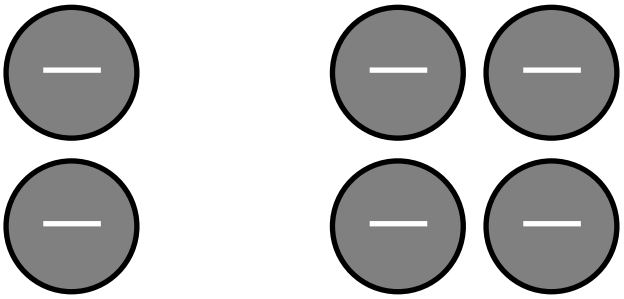


You end up at –6. **So the answer is –2 – 4 = –6.**

You should get the same answer using integer chips. There are actually two ways to solve the problem with chips.

Method 1

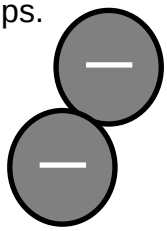
The first way is to think about –2 – 4 as two negative chips plus four more negative chips, or –2 + –4.



You can see that there are 6 negative chips all together. **So, the answer is –6.**

Method 2

The second way to think about –2 – 4 is to start with two negative chips, and take away four positive chips, or –2 – (+4). Here are two negative chips.



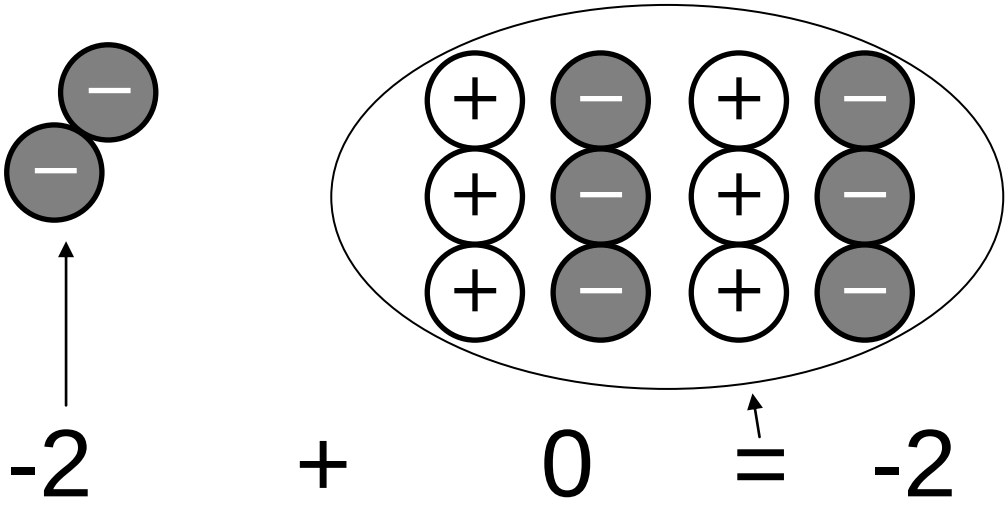
How can you take away 4 positive chips when there are none there?

What if you had a number of positive and negative chips that equaled –2?

Remember:  = 0 and anything plus zero is unchanged.

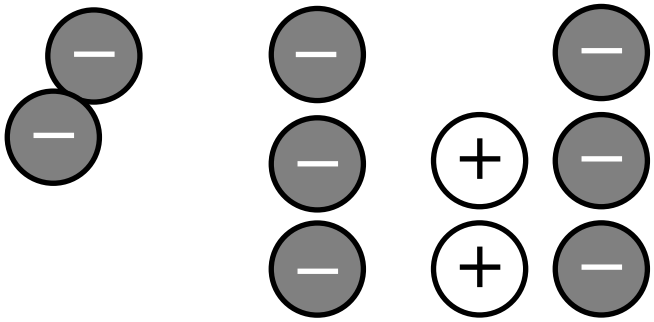
✓ The identity property of addition says that a number plus zero equals the number. Zero does not change the "identity" of the number. For example, 5 + 0 = 5, or $a + 0 = a$.

The identity property will let you add pairs of positive and negative chips, without changing value.

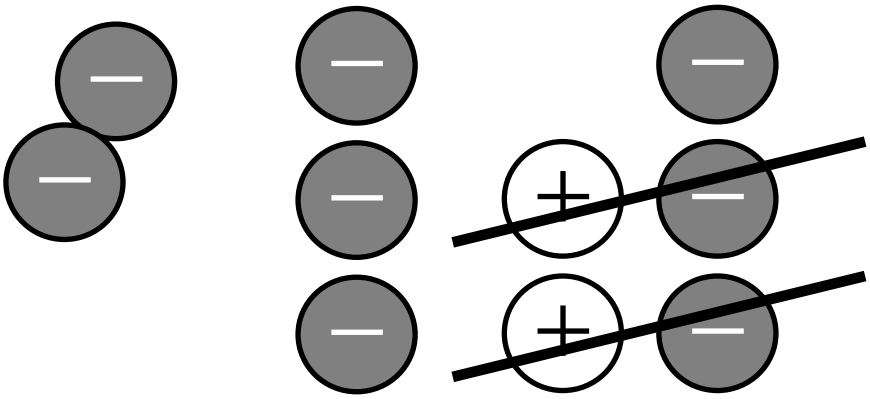


Now you have plenty of chips to use.

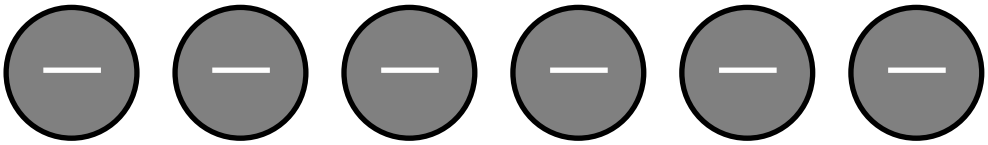
To show –2 – 4 as –2 – (+4), take away four of the positive chips.



Now cancel out groups of 



Count the chips that are left. You can see that there are 6 negative chips.



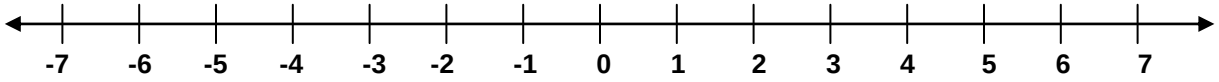
The solution is –6.

The integer chips methods showed that subtraction can be thought of in two ways: as adding negative numbers or as taking away positive numbers!

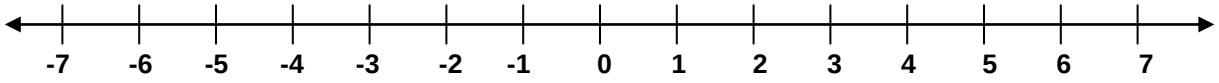
Now you try!

7. Show each of the following using the number line method.

a. –2 + 5



b. 4 – 3



Show these subtraction problems using both integer chip methods (adding negative chips and subtracting positive chips).

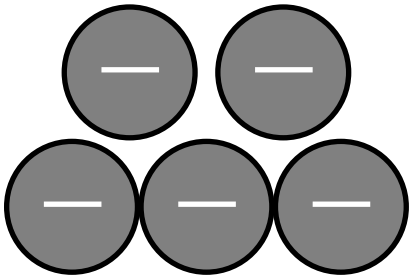
c. 3 – 7

d. 8 – 2

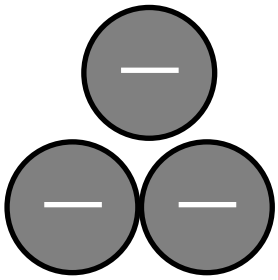
Example: Evaluate –5 – (–2)

Solution: Look at this. You are subtracting a negative number from a negative number! This would be confusing to show with a number line, so let's use our integer chips.

Start with 5 negative chips.



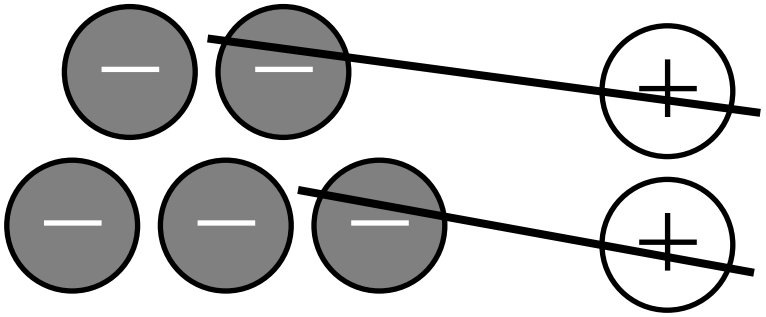
Then, take away 2 negative chips.



You are left with 3 negative chips, or –3.

Parentheses ( ) are sometimes used to make something easier to look at. Here they are used to separate two minus signs.

You just found that –5 – (–2) = –3. What if you evaluate –5 + 2? Using the chips, it will look like the example on the next page:



Cancel groups of plus and minus, because they equal zero.

Again, the answer is –3.

Look at that! Subtracting a negative number is the same as adding a positive number.

$$–5 – (–2) = –5 + 2 = –3$$

Rule to add or subtract integers:

1. Adding a positive integer means add 5 + (+8) = 5 + 8
2. Adding a negative integer means subtract 5 + (–8) = 5 – 8
3. Subtracting a positive integer means subtract 5 – (+8) = 5 – 8
4. Subtracting a negative integer means add 5 – (–8) = 5 + 8

Now you try!

Simplify each expression with only numbers.

9.

a. 3 + 2

b. 6 – 9

c. –4 + (–3)

d. –7 – (–4)

End of Lesson 2