ACReS

Academic & Career Readiness Skills

Two Plus You

Unit 2

National PASS Center
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Genesee Valley Educational Partnership
success stories written here

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Lesson 5

Fractions

Words to know:

- √ **fraction**
- √ **numerator**
- √ **denominator**
- √ **mixed number**

A ***fraction*** compares parts to a whole. It is the quotient of two numbers: **a** and **b**. A fraction can be written $\frac{a}{b}$ and means $a \div b$. The top number of a fraction is called the ***numerator***. The bottom number of a fraction is called the ***denominator***. A mixed number is the sum of a whole number and a fraction. The + sign is not shown.

It looks like this: $3 + \frac{2}{3} = 3\frac{2}{3}$

Imagine you work in a pizza shop. One night, a family of four comes in and orders a pizza. They ask you to cut it into four equal pieces – one for each member of the family.

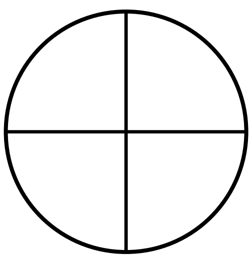
Right after that, a family of five walks in and places the same kind of order. You cut their pizza into five equal slices – one for each member of the family.

Finally, a family of ten walks in, and places the same order! Which pizza will have the biggest slices?

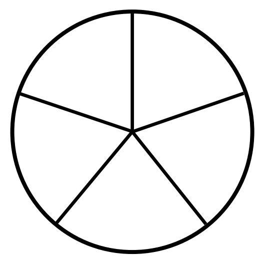
Each pizza ordered was the same size. The first was divided into four equal pieces. The second was divided into five equal pieces. The third was divided into ten equal pieces. Each member of the first family ate more pizza than the members of the other families. Each slice of their pizza was bigger than the slices of the other pizzas. If you cannot see this, don't worry. We can show it with pictures and with math.

Pizza is usually round. Here, a circle represents one whole pizza.

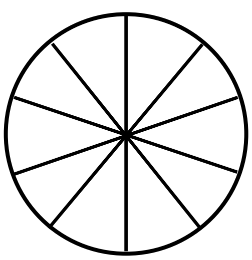
This circle shows the pizza of the family of four. It is divided into four equal parts.



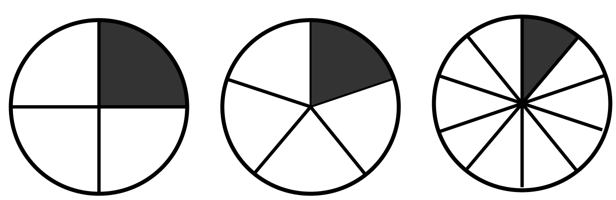
This is the pizza of the family of five. It is divided into five equal pieces.



The pizza of the family of ten is divided into ten equal pieces.



The question asked which family was going to get the biggest slice for each family member. Compare the sizes of the shaded regions of each pizza below.



The pizza cut into four pieces has the largest shaded region. That means that the family of four had the largest slice for each family member.

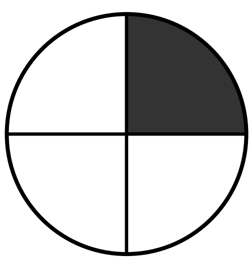
The pizza problem can be shown with math using fractions. Fractions compare the part to the whole.

✓ A fraction is the quotient of two numbers, *a* and *b*. A fraction is written as $\frac{a}{b}$, and it means $a \div b$

Each of the pizzas made for the three different families can be shown using fractions.

Fractions are most often used to represent part of a whole. We would write this as $\frac{\textit{part}}{\textit{whole}}$

The pizza of the family of four is shown below.



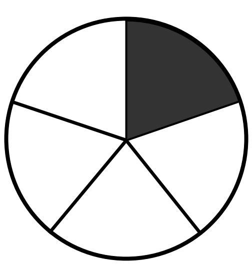
The whole pizza is made of 4 slices.

One slice is 1 part of the whole pizza.

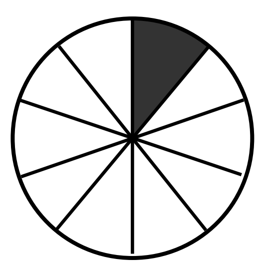
One slice is one of four slices.

This idea can be shown by the fraction, $\frac{1}{4}$

The pizzas of the other families can be shown like this:



$= \frac{1}{5}$



$= \frac{1}{10}$

The shaded area in each drawing is a slice of pizza. It is 1 part of the whole pizza. The whole pizza is made up of the total number of slices. The slice or part is the numerator of a fraction. The total number of parts, or the whole, is the denominator.

- ✓ The top number of a fraction is called the ***numerator***.
- ✓ The bottom number is called the ***denominator***.

$$\frac{\textbf{numerator}}{\textbf{denominator}}$$

The denominator tells you the total number of pieces in the whole. The numerator tells you how many of those pieces you have.

Example: $\frac{1}{2}$ means one out of two pieces, or one-half;

$\frac{1}{3}$ means one out of three pieces, or one-third;

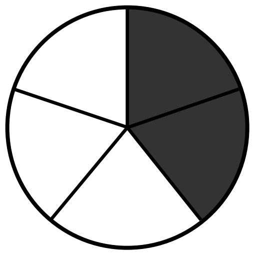
$\frac{1}{4}$ means one out of four pieces, or one-fourth;

$\frac{1}{5}$ means one out of five pieces, or one-fifth, and so on.

The pizza problem focused on 1 individual slice. What happens when you are dealing with multiple slices?

Example: Two people in the family of five decide to save their slices of pizza for later. What fraction of the whole pizza is saved for later?

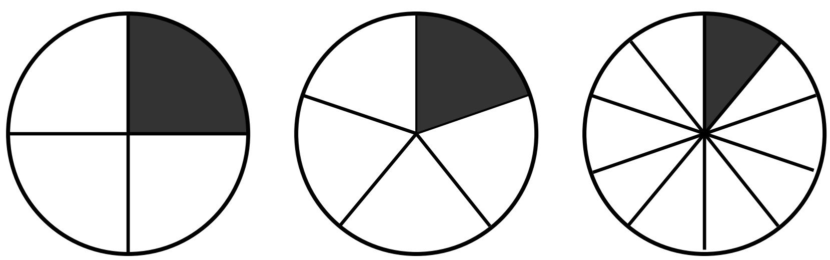
***Solution*:** In this case, there are 5 total slices. Two (2) of them are saved. The denominator will be 5 because there are 5 total slices. The numerator will be 2 because you are focusing on 2 out of 5 slices.



$$= \frac{2}{5}$$

So, $\frac{2}{5}$ of the pizza is saved for later.

Go back to the initial pizza problem.



$\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{10}$

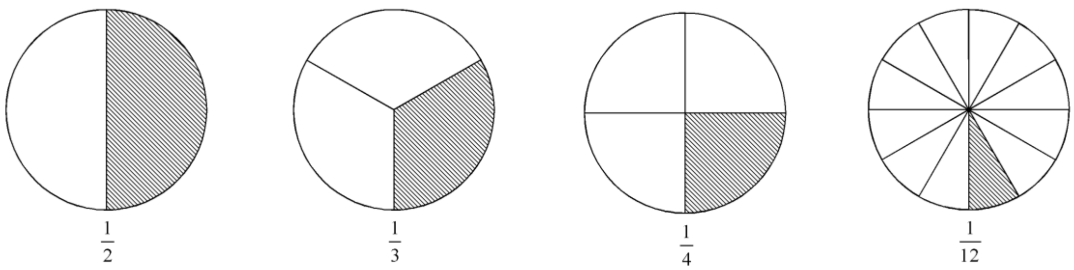
The shaded region of the circle on the left is the largest. The shaded region of the circle on the right is the smallest. You can compare the shaded regions using fractions. It would look like this:

$$\frac{1}{4} > \frac{1}{5} > \frac{1}{10}$$

You can use this method for comparing fractions only if each fraction is part of the same whole object. In this case, identical circles represented one whole pizza.

Example: Compare the size of the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{12}$. Put them in order, from least to greatest.

***Solution*:** We will use pictures to help us answer this question.

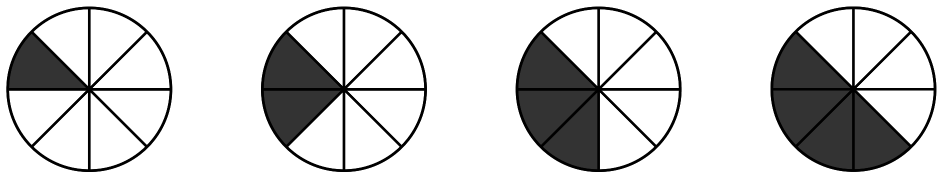


In the pictures above, the same size circle represents one whole. Each circle is divided into equally sized pieces. You can see that more pieces = smaller size. The order of the fractions, from least (smallest) to greatest (largest), is

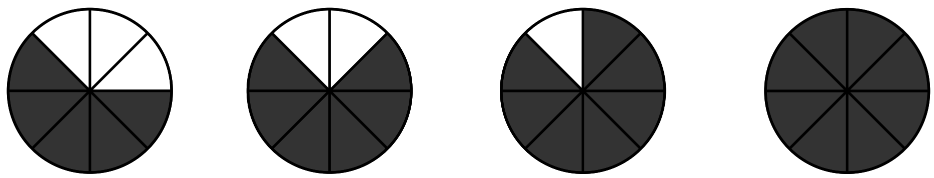
$$\frac{1}{12} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{2}$$

✓ If a set of fractions has the same numerator, the fraction with the smallest denominator is largest in value.

What if fractions have the same denominators, but <u>different numerators</u>? Look at the example below. What do you notice?



$\frac{1}{8}$ $\frac{2}{8}$ $\frac{3}{8}$ $\frac{4}{8}$



$\frac{5}{8}$ $\frac{6}{8}$ $\frac{7}{8}$ $\frac{8}{8}$

As the numerator grows, so does the value of the fraction. Why is this? Remember: The numerator tells you how many pieces of the whole the fraction has. The fraction $\frac{1}{5}$ tells you have one piece of a whole with five pieces, or one-fifth. How does this compare to the fraction $\frac{2}{5}$?

You still have fifths, but now there are two of them.

$\frac{2}{5}$ must be bigger than $\frac{1}{5}$.

Try some fraction problems on your own.

1. Circle which fraction is larger.

a. $\frac{1}{11}$ or $\frac{1}{9}$ **b.** $\frac{6}{17}$ or $\frac{6}{15}$ **c.** $\frac{13}{19}$ or $\frac{11}{19}$

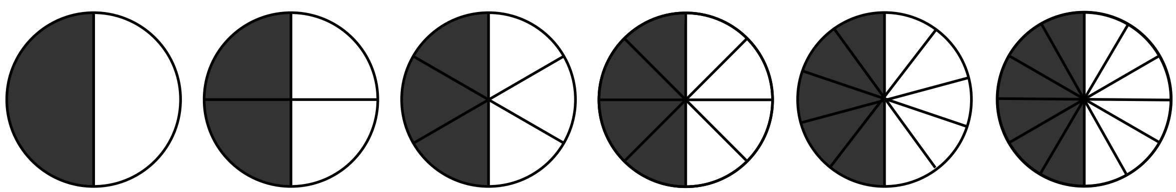
Comparing negative and positive fractions works the same way as comparing negative and positive integers.

Example: Think about $-\frac{2}{3}$ and $\frac{1}{3}$. The first fraction might seem bigger, because its numerator is 2. But, $-\frac{2}{3}$ is a negative number and less than 0. One-third ($\frac{1}{3}$) is a positive number and greater than 0.

$$-\frac{2}{3} < \frac{1}{3}$$

Equivalent Fractions

Sometimes, fractions that look different may be equal in value. Look at the picture form of these fractions.



$\frac{1}{2}$ $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$ $\frac{5}{10}$ $\frac{6}{12}$

You can see that the shaded portions of the circles all equal each other. They are all one half of the circle. The only difference is the number of equal pieces each circle has.

You can use algebra to show why the fractions are all equivalent – have equal value. Once again, the equal forms of $\frac{1}{2}$ are $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, and, $\frac{6}{12}$ ...

Do you see a pattern with the numbers? Let’s rewrite the fractions to show more clearly how they are equal.

$$\frac{1}{2} = \frac{1\times1}{2\times1} = \frac{1}{2}\left(\frac{1}{1}\right)$$

$$\frac{2}{4} = \frac{1\times2}{2\times2} = \frac{1}{2}\left(\frac{2}{2}\right)$$

$$\frac{3}{6} = \frac{1\times3}{2\times3} = \frac{1}{2}\left(\frac{3}{3}\right)$$

$$\frac{4}{8} = \frac{1\times4}{2\times4} = \frac{1}{2}\left(\frac{4}{4}\right)$$

$$\frac{5}{10} = \frac{1\times5}{2\times5} = \frac{1}{2}\left(\frac{5}{5}\right)$$

$$\frac{6}{12} = \frac{1\times6}{2\times6} = \frac{1}{2}\left(\frac{6}{6}\right)$$

FACT

Any number divided by itself equals one. For example $\frac{5}{5} = 1$ *and* $\frac{z}{z} = 1$

Each equal form of $\mathbf{\frac{1}{2}}$ is simply $\mathbf{\frac{1}{2}}$ multiplied by something equal to one!

Each fraction above can be written as $\frac{1}{2}\times1$. The identity property says that $\frac{1}{2}\times1=\frac{1}{2}$. Thus, every fraction above equals $\frac{1}{2}$.

FACT

The <u>identity property of multiplication</u> states that any number multiplied by 1 is that number. It allows you to multiply any fraction by 1 without changing its value!

You can use the identity property to find equivalent fractions for *any* fraction.

Example: Write two fractions that are equivalent to $\frac{1}{3}$.

Solution: If you multiply both the numerator and denominator by 2, you will see that

$$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$$

Or you can multiply both the numerator and denominator by 3 and get

$$\frac{1}{3} = \frac{1 \times 3}{3 \times 3} = \frac{3}{9}$$

Thus, $\frac{2}{6}$ and $\frac{3}{9}$ are equivalent to $\frac{1}{3}$.

Now you try!

2. Write two fractions that are equivalent to:

a. $\frac{3}{5}$

b. $\frac{2}{3}$

3. Complete the equivalent fraction.

a. $\frac{3}{5} = \frac{12}{\square}$

b. $\frac{16}{24} = \frac{\square}{12}$

Equivalent fractions can be useful. They can help you rewrite fractions using smaller numbers.

Example: $\frac{30}{45}$ is a fraction with large numbers. You can simplify it using equivalent fractions and common factors.

✓ A fraction is in simplest form or lowest terms, if the numerator and denominator share no common factors. Thus, the fraction has no equivalent forms with smaller numbers.

Simplest form and lowest terms mean the same thing. To "simplify" means to put something into simplest form or lowest terms.

Example: For instance, $\frac{6}{9}$ is not in lowest terms. The numerator and denominator share a common factor of 3.

$\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$ Now it is in lowest terms.

Rule to put a fraction in lowest terms:

1. Find the Greatest Common Factor of the numerator and denominator.

2. Divide the numerator and denominator by that factor.

Remember: If the numerator and denominator of a fraction share no factors, it is in its simplest form. If a fraction is in simplest form, the GCF of its numerator and denominator is 1.

Example: Write $\frac{30}{45}$ in lowest terms.

Solution

Steps 1 & 2: Write the factors of the numerator and denominator. Underline, circle, or highlight the factors they have in common.

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30
Factors of 45: 1, 3, 5, 9, 15, 45

Step 3: You can see that fifteen is the greatest common factor.

Step 4: Divide the numerator and denominator by the GCF.

$$\frac{30}{45} = \frac{30 \div 15}{45 \div 15} = \frac{2}{3}$$

Now you try!

4. Write the following fractions in simplest form.

a. $\frac{4}{12}$ **b.** $\frac{6}{15}$ **c.** $\frac{4}{5}$

You can use the cross product method to check if fractions are equivalent.

Cross Product Method

- ✓ Multiply the numerator of the first fraction by the denominator of the second fraction.
- ✓ Then, multiply the denominator of the first fraction by the numerator of the second fraction.
- ✓ If these two products are equal, the fractions are equivalent.

If $\frac{1}{2} = \frac{2}{4}$, then $(1 \times 4) = (2 \times 2)$. If $\frac{a}{b} = \frac{c}{d}$, then $(a \times d) = (b \times c)$.

It is called the cross product method, because you multiply <u>across</u> the equals sign as shown below.

Example: Show that $\frac{1}{4} = \frac{3}{12}$ using the cross product method.

***Solution*:** Write the fraction equation.

Multiply 1 x 12 and then, 4 x 3, as shown.

$$\frac{1}{4} = \frac{3}{12}$$

$$(1 \times 12) = (3 \times 4)$$

$$12 = 12 \ \checkmark$$

The cross products are equal.

The fractions are equivalent.

Now you try!

5. Which fraction is *<u>not</u>* equivalent to $\frac{2}{3}$? (Circle the correct answer.)

a. $\frac{2}{4}$ **b.** $\frac{6}{9}$ **c.** $\frac{4}{6}$ **d.** $\frac{20}{30}$

6. Decide whether each fraction is in simplest form. Simplify any fraction that is not.

a. $\frac{8}{16}$ **b.** $\frac{12}{18}$ **c.** $\frac{9}{10}$ **d.** $\frac{13}{64}$

Mixed Numbers

Fractions represent parts of whole numbers. They are often combined with whole numbers in everyday life. A whole number plus a fraction is called a mixed number.

✓ A mixed number is the sum of a whole number and a fraction. When written, the addition sign is still there, but it is hiding.

$$3+\frac{2}{3} = 3\frac{2}{3}, \text{ and } A+\frac{b}{c} = A\frac{b}{c}.$$

Example: A grape picker picks enough grapes to fill three large barrels and $\frac{2}{3}$ of a fourth barrel. This can be shown by the model below.

$$1+1+1+\frac{2}{3}$$

You can see that $1+1+1+\frac{2}{3}=3+\frac{2}{3}$.

$$3+\frac{2}{3} = 3\frac{2}{3}.$$

Mixed numbers are not whole numbers. They are between two whole numbers. In the example above, the grape picker picked three barrels of grapes plus part of a fourth barrel. So, the mixed number $3\frac{2}{3}$ is between the whole numbers 3 and 4.

There is a specific way to say mixed numbers. You say $3\frac{2}{3}$ as "three and two-thirds".

The mixed number, $5\frac{3}{4}$, is said as, "five and three-fourths".

Notice: the word "and" comes between the whole number and the fraction.

Mixed numbers can be made into fractions.

Use the definition and work backwards.

$$3\frac{2}{3} = 3 + \frac{2}{3}$$

Now, break up the 3.

$$= 1 + 1 + 1 + \frac{2}{3}$$

Substitute the equivalent fraction for each 1 – three-thirds.

$$= \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{2}{3}$$

$$= \frac{9}{3} + \frac{2}{3}$$

You can add the numerators because the denominators are the same – 3.

$$= \frac{11}{3}$$

✓ If the numerator of a fraction is less than its denominator, it is called a *proper fraction*. If the numerator of a fraction is greater than or equal to its denominator, it is called an *improper fraction*.

For instance, $\frac{13}{3}$ is an improper fraction because the top, 13, is bigger than the bottom, 3. The fraction $\frac{7}{18}$ is a proper fraction, since 7 is smaller than 18.

Mixed numbers and whole numbers can always be shown as improper fractions.

When $3\frac{2}{3}$ was changed into an improper fraction, you saw that $3\frac{2}{3}$ really meant "three wholes and two thirds". Then, three wholes was changed to the equivalent nine-thirds. Finally, nine-thirds was combined with two-thirds to get the answer, $3\frac{2}{3} = \frac{11}{3}$.

This method can be used to convert any mixed number into an improper fraction.

Rule to convert a mixed number to an improper fraction:

1. Multiply the whole number part by the denominator of the fraction.
2. Add this product to the numerator of the fraction.
3. Put the fraction into simplest form.

$$5\frac{3}{4} = 5\frac{3}{4} = \frac{(5 \times 4) + 3}{4} = \frac{23}{4}$$

Now you try!

7. Write the mixed numbers as improper fractions.

a. $1\frac{1}{3}$ **b.** $2\frac{7}{8}$ **c.** $3\frac{3}{4}$ **d.** $5\frac{3}{5}$

What if you have an improper fraction and want to make it into a mixed number?

Example: Write $\frac{9}{4}$ as a mixed number.

Solution: In this example, you are working with fourths. You already know that there are four fourths in every whole, because $\frac{4}{4}=1$.

The numerator of the fraction tells you how many pieces you have. In this case there are nine pieces of fourths. What you need to know is how many groups of four are in nine. To find that, divide nine by four.

> *Think Back*
>
> *Fractions mean division.*
>
> $\frac{9}{4}=9\div 4$

$$\begin{array}{r} 2 \\ 4\overline{)9} \\ -\underline{8} \\ 1 \end{array}$$

There are two wholes and one remaining fourth. The answer is

$$\frac{9}{4} = 2\frac{1}{4}.$$

You can use this method to convert any improper fraction to a mixed number.

Rule to convert an improper fraction to a mixed number:

1. Divide the fraction's numerator by its denominator.
2. The number of times it divides evenly is the "whole" part of the mixed number.
3. To the right of that, write a fraction. The numerator will be the <u>remainder</u> found in step 2. The denominator will be the same as that of the original fraction.

$\frac{9}{2}$

$=9\div 2$

$=4\,R\,1$

$=4\frac{1}{2}$

Example: Write $\frac{14}{3}$ as a mixed number.

Solution

First write $\frac{14}{3}$ as a division problem. $3\overline{)14}$

Find the quotient with the remainder.

$$\begin{array}{r} 4\ R\ 2 \\ 3\overline{)14} \\ -12 \\ \hline 2 \end{array}$$

The 4 stays on the left as the whole number.

The 2 becomes the numerator of the fraction.

The 3 becomes the denominator of the fraction.

$$4\frac{2}{3}$$

8. Write the improper fractions as mixed numbers in simplest form.

a. $\frac{5}{3}$

b. $\frac{21}{8}$

c. $\frac{5}{4}$

d. $\frac{11}{5}$

End of Lesson 5