ACReS

Academic & Career Readiness Skills

Two Plus You

Unit 2

National PASS Center
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Lesson 6

Decimals

Words to know:

- √ decimal
- √ terminating decimal
- √ repeating decimal
- √ rounding

A ***decimal*** can represent a whole number or the fractional part of a number. Let's take a closer look at decimals.

The price of bananas at your local grocery store is ten bananas for one dollar. You can show the price using a fraction. One banana costs

$1 \div 10 = \frac{1}{10}$ of a dollar.

You know that one tenth of one dollar is a dime, or $0.10. So,

$$\frac{1}{10} = .10$$

Here is another way to represent fractions. 0.10 is an example of a decimal.

✓ A decimal is a number that can represent a whole part or a fractional part. A period (.) separates the whole part from the fractional part. It is known as a decimal point. For example, 3.5 is a decimal and so is 0.72.

A good way to understand decimals is to think of them as money. The first number to the right of a decimal point is in the <u>tenths</u> place. With money, one dime is $0.10, or one-<u>tenth</u> of a dollar. The second number to the right of a decimal point is in the <u>hundredths place</u>. With money, this number tells you the number of pennies you have. One penny is 1 one-hundredth ($\frac{1}{100}$) of a dollar. Or, 100 pennies equal one dollar. The diagram below shows whole number and decimal places. Numbers to the left of the decimal point are whole numbers. Numbers to the right of the decimal point are fractions.

___	___	___ ,	___	___	___	●	___	___	___	___	___	___
hundred thousands	**ten thousands**	**thousands**	**hundreds**	**tens**	**ones**		**tenths**	**hundredths**	**thousandths**	**ten thousandths**	**hundred thousandths**	**millionths**

Example: Write the place value of each digit in the number **.**123450

Solution: 1 is in the tenths place. 2 is in the hundredths place. 3 is in the thousandths place. 4 is in the ten-thousandths place. 5 is in the hundred-thousandths place. Zero is in the millionths place. You can also say that there is 1 tenth, 2 hundredths, 3 thousandths, 4 ten-thousandths, 5 hundred-thousandths, and zero millionths.

Now you try!

1. Write each digit in the correct place value in the chart below. Then write the place value of the digit that is farthest to the right.

a. ______________________________

b. ______________________________

c. ______________________________

d. ______________________________

e. ______________________________

a.			•					
b.			•					
c.			•					
d.			•					
e.			•					
	Tens	**Ones**	•	**Tenths**	**Hundredths**	**Thousandths**	**Ten-Thousandths**	**Hundred-Thousandths**

How do you write and say the entire decimal?

Example: Write out 3.413 using words.

Solution: This is a mixture of whole numbers and a fractional part. By counting the number of decimal places, you can see that this number goes to the thousandths place. Say it like this:

3.413 = three and four hundred thirteen thousandths.

- ✓ The whole number is said the way it usually is said.
- ✓ "And" means that there is a decimal point there.
- ✓ The numbers after "and" are read as a fraction. The place value of the digit farthest right is the denominator. In the above example, it is thousandths.

Decimals are read the way mixed numbers are read. Without much work, decimals can be written as mixed numbers.

$$3.413 = 3\frac{413}{1000}$$

Once again, the number is read as three and four hundred thirteen thousandths.

Using mixed numbers, you can change decimals into improper fractions, too!

$$3\frac{413}{1000} = \frac{3000}{1000} + \frac{413}{1000} = \frac{3413}{1000}, \quad \text{so} \quad 3\frac{413}{1000} = \frac{3413}{1000}.$$

Rule to write a decimal with words:

1. Write the number to the left of the decimal as you would any whole
2. number.
3. In place of the decimal point, write the word “and”.
4. Write the number to the right of the decimal point, as you would any whole number.
5. At the end, write the final digit’s place value. It should end in “ths.” (tenths, hundredths, thousandths, …)

Rule to write a decimal as a mixed number:

1. Rewrite all the digits to the left of the decimal point. This is the whole
2. number part.
3. Write all the digits to the right of the decimal point as the numerator of a fraction.
4. For the denominator, write the place value of the right-most digit. (10, 100, 1000, 10000, 100000, …)

For example, $17.927 = 17\frac{927}{1000}$

Now you try!

2. Write each decimal using words, then as a mixed number, then as a fraction in lowest terms.

a. 2.6

Words: ____

Mixed Number:

Fraction:

b. .43

Words: ____

Mixed Number:

Fraction:

c. 1.6524

Words: ____

Mixed Number:

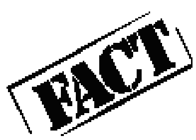
Fraction:

Consider two whole numbers, 340 and 00340. Believe it or not, 340 = 00340. The number 00340 looks strange. Numbers are not usually written this way. The first two zeros before the 3 have no meaning. However, the zero after the 4 is needed. If you drop the zero at the end of 340, the value of the number changes.

Similar things can be done with decimals. The following decimals are all equal.

$$
\begin{aligned}
&0.43\\
&= 0.430\\
&= 0.4300\\
&= 0.43000\\
&= 0.430000\\
&= 0.43000000000000000000
\end{aligned}
$$

They are the same because the place value of the 4 and 3 never change.



Any number of zeros may be added ***at the end of a decimal*** *without changing the value of the decimal.*

Now you try!

2. True/False. Decide whether each equation is true or false. Put "T" or "F" on the lines provided.

a. ____ 07 = 7

b. ____ 4 = 40

c. ____ 00030 = 00300

d. ____ 3.4 = 03.4

e. ____ 8.42300 = 8.423

f. ____ 900.163200 = 0900.1632

Knowing this helps us put decimals in order.

Example: Which is larger, .2 or .19?

Solution: You might think that .19 is larger than .2, since 19 > 2. But think about this first.

You know that .2 = .20

In terms of money, you also know that $0.20 is more money than $0.19.

Therefore, .2 > .19 .

What about the next two decimals?

Example: Which is larger, 0.2 or .199999999999999999?

Solution: Line up the two numbers according to their place values.

.2

.199999999999999999

Notice that the top number has 2 tenths, and the bottom has only 1 tenth plus something that is less than one tenth, so:

.2 > .199999999999999999

Decimals are helpful when comparing the size of two numbers. That is why they are used for money instead of fractions. What you have seen here will help you use the next method of comparing the size of two decimals.

Rule to compare the size of decimals:

1. Line up the two decimals according to place value. An easy way to do this is to make sure the decimal points are on top of each other.

2. Compare place values until a difference is found. Start with the whole number parts. If those are the same, check the tenths place of each. If they are the same, check the hundredths, then the thousandths, etc. Keep checking until you find a place value where the digits are not the same.

3. Determine which is larger.

In the place value where you find the difference, the larger digit tells you which number is larger.

Example: Compare 1.1324549 and 1.1324639.

Solution

Step 1: Line up the two numbers by their decimal points.

$$\begin{array}{l} 1.1324549 \\ 1.1324639 \end{array}$$

Step 2: Compare place values until a difference is found. Circle the difference. Notice that it is in the hundred-thousandths place.

Step 3: Determine which is larger. You circled the digits 6 and 5. $6 > 5$, so $1.1324639 > 1.1324549$.

Now you try!

3. Compare the following decimals using < , > , or =.

a. 12 and .13

b. 102 and .13

c. and .999

d. 16.82736 and 16.82747

Terminal and Repeating Decimals

Decimals are created by performing the division in fractions. Divide the denominator of a fraction into its numerator. The result will be a decimal.

Example: Look at the fraction, $\frac{1}{2}$. When division is performed, it becomes the decimal .5.

$$1 \div 2$$

$$\frac{1}{2} = \qquad = 2\overline{)1}$$

$$\begin{array}{r} . \\ 2\overline{)1.0} \end{array}$$

Make 1 into 1.0, and put a decimal point directly above the division bar too.

$$\begin{array}{r} 0. \\ 2\overline{)1.0} \\ \underline{-0} \\ 1 \end{array} \qquad \begin{array}{r} 0. \\ 2\overline{)1.0} \\ \underline{-0} \\ 10 \end{array} \qquad \begin{array}{r} 0.5 \\ 2\overline{)1.0} \\ \underline{-0} \\ 10 \\ \underline{-10} \\ 0 \end{array}$$

Now just divide, as if it's the whole number 10.

Example: Write $\frac{5}{8}$ as a decimal.

Solution $\frac{5}{8} = 5 \div 8$

$$8\overline{)\,5}$$

$$\begin{array}{r} \mathbf{0.} \\ 8\overline{)\,\mathbf{5.0}} \\ -\underline{\mathbf{0}} \\ \mathbf{5} \end{array} \qquad \begin{array}{r} \mathbf{0.} \\ \overline{)\,\mathbf{5.0}} \\ -\underline{\mathbf{0}} \\ \mathbf{50} \end{array} \qquad \begin{array}{r} 0.6 \\ 8\overline{)\,5.0} \\ -\underline{0} \\ 50 \\ -\underline{48} \\ 2 \end{array} \qquad \begin{array}{r} 0.6 \\ 8\overline{)\,5.00} \\ -\underline{0} \\ 50 \\ -\underline{48} \\ 20 \end{array}$$

If there is a remainder, create a decimal and keep adding zeros to the dividend until there is no remainder.

$$\begin{array}{r} 0.625 \\ 8\overline{)\,5.000} \\ -\underline{0} \\ 50 \\ \underline{48} \\ 20 \\ -\underline{16} \\ 40 \\ -\underline{40} \\ 0 \end{array}$$

In both of the above examples, the resulting decimals had an end. Decimals that have an end are called terminating decimals. Not all decimals are terminating. Not all of them have an end. Take a look at the example on the next page.

Example: A sign in a store says "*Markers: 3 for $1.00 or 1 for $ .35*". Which one is the better deal?

Solution: You must figure out what the cost of one marker is in each deal and compare the prices. In the first deal, 3 markers are offered for one dollar. The price of one marker can be shown as $\frac{1}{3}$ of a dollar. How much is this? Remember: Fractions mean division.

$$\frac{1}{3} = 1 \div 3 \quad = 3\overset{\displaystyle 0.3333}{\overline{)1.0000}}$$

$$\begin{array}{l} \underline{.9} \\ .10 \\ \underline{.09} \\ .010 \\ \underline{.009} \\ .0001 \end{array}$$

No matter how long you keep dividing, this decimal will never end. You will keep adding on another 3 forever! A decimal that does not end is called a repeating decimal.

To show that a decimal never ends put a bar over the part that repeats. In the example above, 0.33333333333333… is written as $.\overline{3}$. Because you are dealing with money, you need to round this decimal to the hundredths place. (Rounding will be explained later in this lesson.)

.3333333. . . or $.\overline{3}$ rounds to .33.

Thus, one marker from the first deal will cost $.33. That is a cheaper price than the $.35 of the other deal.

Another example of a decimal that never ends is 0.6437121212121212121212…

It is written as $0.643\overline{12}$. Notice that the bar only goes over the numbers that repeat.

That makes the decimal easier to read.

- ✓ A decimal that ends is called a **terminating decimal**. For instance, .173 and 33.2 are terminating decimals.
- ✓ A **repeating decimal** is a decimal that has an infinite number of digits. The digits continue in a set pattern.
- ✓ For example, $.3333333... = .\overline{3}$, and $.473473473473473... = .\overline{473}$ are repeating decimals.

Any fraction can be made into either a terminating or a repeating decimal!

Example: Write $\frac{4}{5}$ as a decimal.

Solution: Use long division.

$$\begin{array}{r} 0.8 \\ 5\overline{)\ 4.0} \\ -\underline{4.0} \\ 0 \end{array}$$

$\frac{4}{5} = 0.8$, a terminating decimal.

Example: A sign in the store reads "***Paper towels, 11 rolls for $3.00***". How much will 1 roll of paper towels cost?

Solution: You must divide 3.00 into 11 equal groups.

$$
\mathbf{3 \div 11 =} \quad
\begin{array}{r}
0.27272...\\
11\overline{)3.00000...}\\
-\underline{2.2}\\
80\\
-\underline{77}\\
30\\
-\underline{22}\\
80\\
-\underline{77}\\
\ddots
\end{array}
$$

As soon as you see the same remainder twice, you know that the decimal is a repeating decimal. You can stop dividing at that point.

The answer: $3 \div 11 = .\overline{27}$

Rounding

In the previous example, you found that one roll of paper towels cost $\$0.\overline{27}$. A repeating decimal is acceptable in mathematics. It does not work for money. Stores cannot charge $0.272727272727. . . for an item. If $0.27 equals 27 pennies, how many pennies equal $0.27272727? Pennies are the smallest division of a dollar. Parts of a penny do not exist.

When dealing with money, repeating decimals are rounded to the nearest hundredth or cent. One one-hundredth of a dollar = one cent = one penny. In the case of the paper towels, $0.27272727 is rounded to $0.27. Thus, the cost of one roll of paper towels is $0.27.

Rule to round a number to a given place value	
1. Look at the number to the right of the place value you are asked to round to.	Round 1.895 to the nearest hundredth.
2. Compare that number to 5.	
a. If that number is less than 5, round down and leave the given place value the same.	1.895
b. If that number is greater than or equal to 5, round up and increase the given place value by 1.	5 = 5 Round up
c. If the number in the given place value is a 9, make it a 0 and increase the value of the number to the left of our given place value by 1.	1.895 rounds to 1.90
d. In equations, rounded answers require a special sign. Use **≈**, not = in the equation.	

Example: Round 173.9378429329 to the nearest tenth.

Solution

Step 1: Look at the number to the right of the tenths place.

173.9**3**76429329

Step 2: Compare the number to 5. Notice that $3 < 5$, so you must round down. Leave the number in the tenths place the same. Your answer is 173.9.

Now you try!

Convert the following fractions to decimals. The decimals may be either terminating or repeating.

4. $\frac{9}{11}$

5. $\frac{11}{8}$

6. $\frac{5}{6}$

Convert the following fractions to decimals. The decimals may either terminate or repeat. Then, round each decimal to the nearest <u>hundredth</u>.

7. $\frac{3}{8}$

8. $\frac{2}{3}$

9. $\frac{5}{11}$

10. Write the decimal with words, then as a mixed number, then as an improper fraction in simplest form.

4756.5

11. Use an inequality sign (> or <) to compare each pair of decimals.

a. 3.425 ____ 6.425 **b.** 1.089 ____ 1.1

c. 0.001 ____ 0.01 **d.** 142.284756 ____ 142.284755

12. Round each decimal to the nearest <u>hundredth</u>.

a. 7.43232 **b.** 14.267239 **c.** 9.473

d. 1.1111111111 **e.** 0.9877654 **f.** $13.\overline{8}$

13. Write the following decimals so that their place values are lined up.

24971894781.34 and 32.823743239

14. Write the amount as a decimal part of a dollar. (Hint: think of how many <u>cents</u> each equals.)

a. 1 quarter $ _____ **b.** 4 nickels $ _____ **c.** 89 pennies $ _____ **d.** 14 dimes $ _____

End of Lesson 6